

fig. 4.1. Two solutions of equation $x'' = f(x) \in C^1(\mathbb{R})$ showed in fig. (a) are possible, in fig. (b) and fig (c) - impossible.

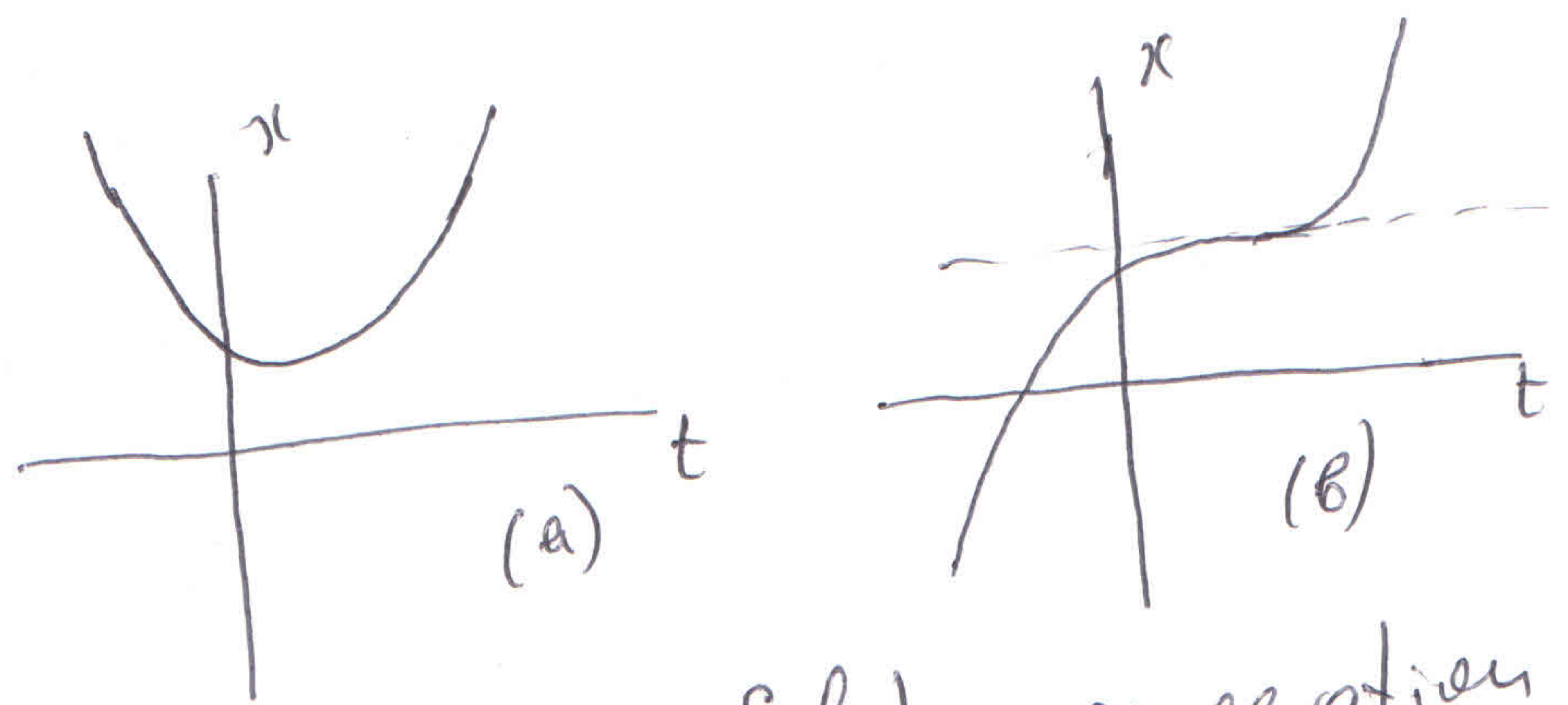


fig. 4.2. Solution of equation $x'''' = f(x) \in C^1(\mathbb{R})$ showed in fig. (a) is possible, in fig. (b) - impossible.

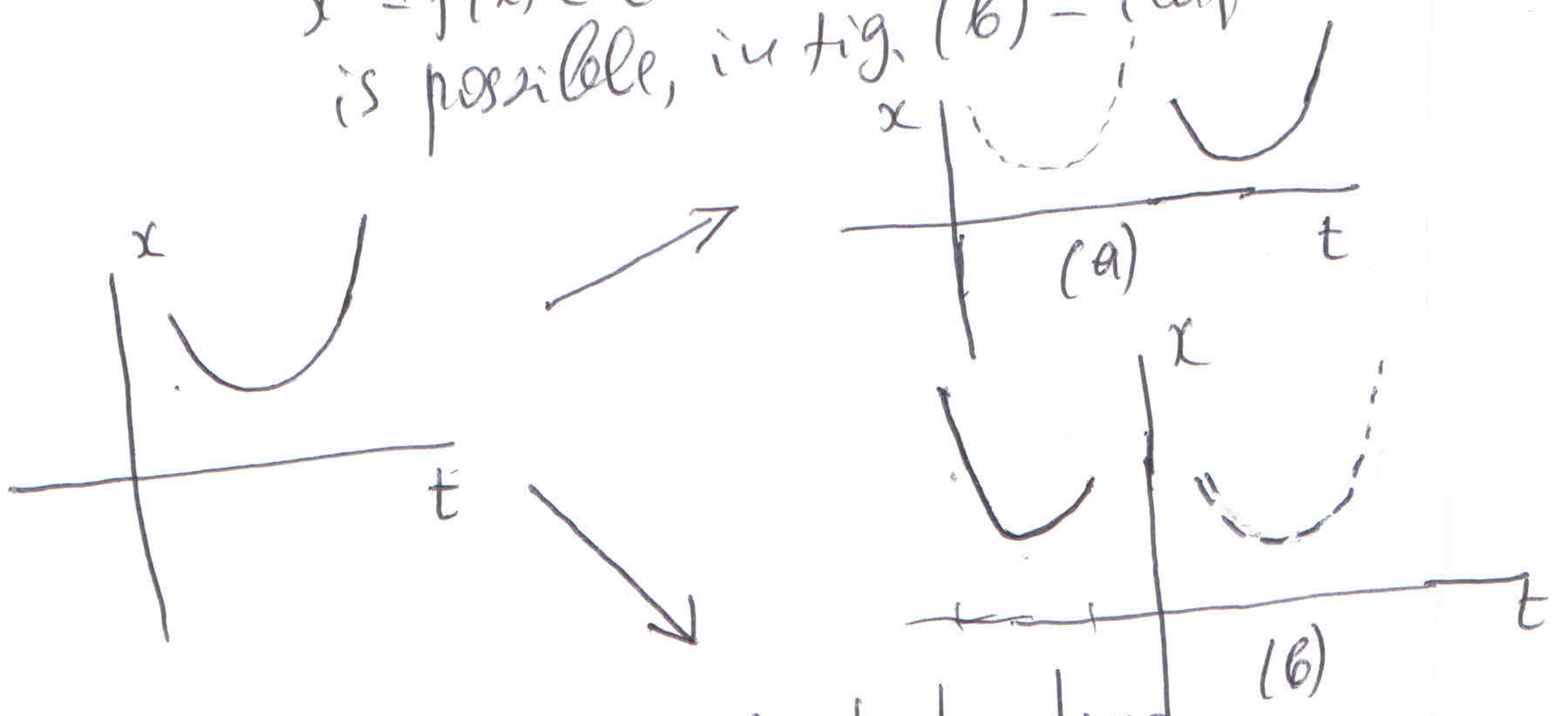


fig. 4.3. (a): shift of time
(b): inverse of time

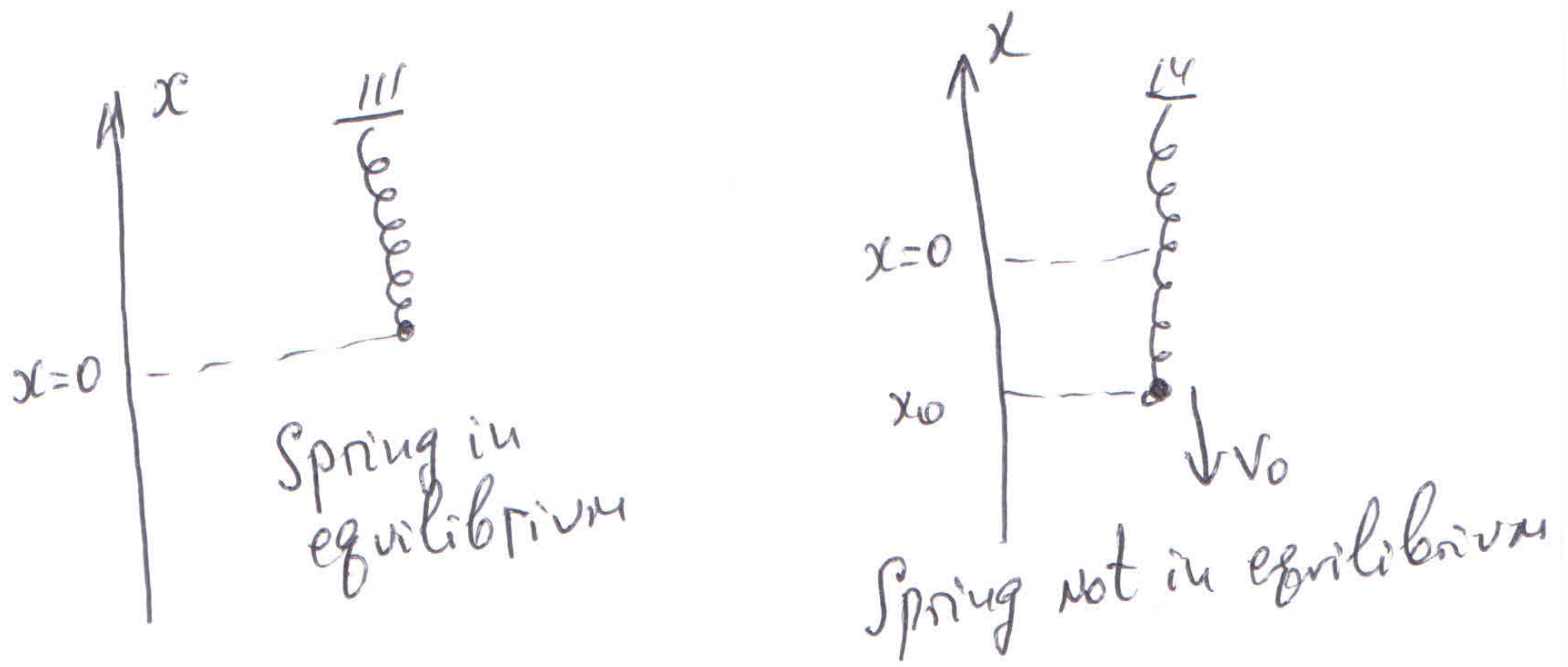


fig. 4.4. The spring

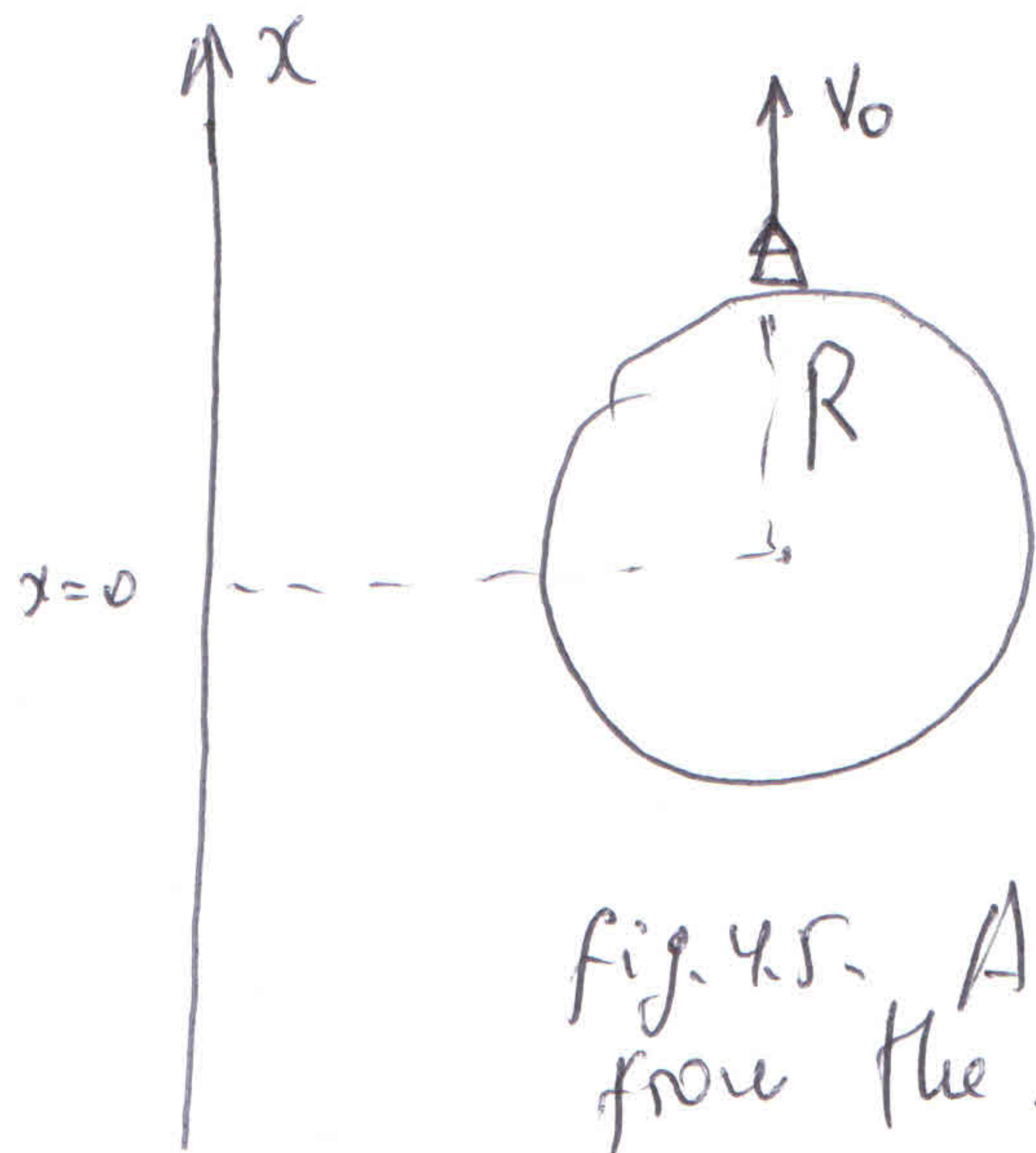


fig. 4.5. A rocket launched up from the surface of the Earth

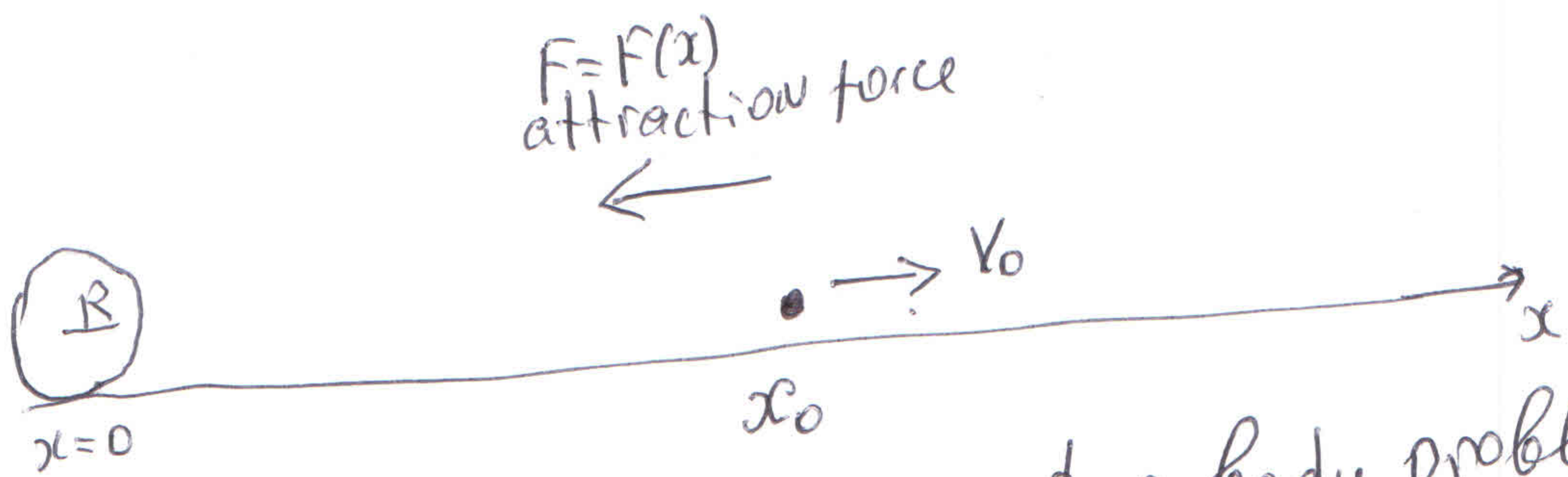


fig. 4.6 One of two body problems.

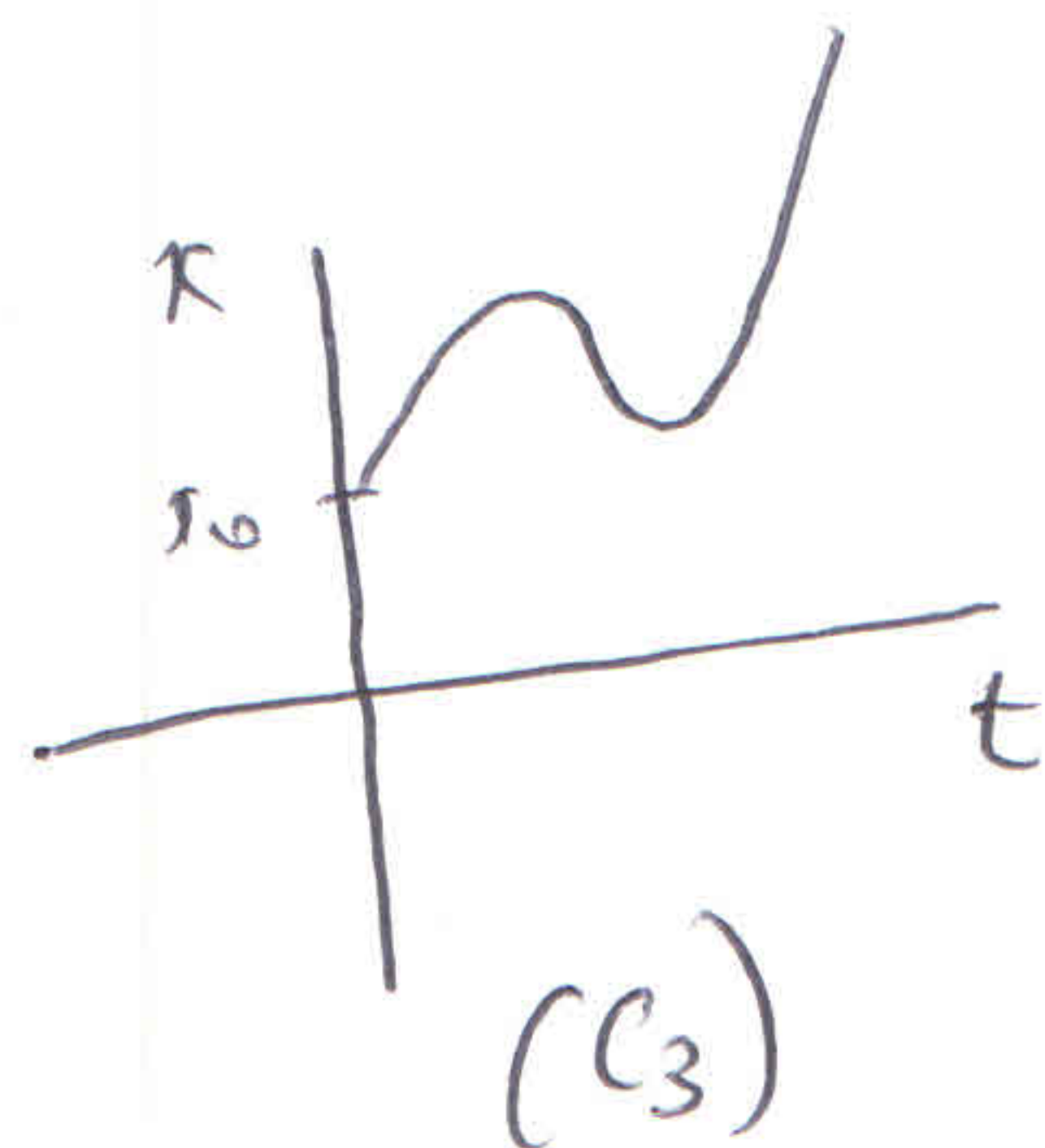
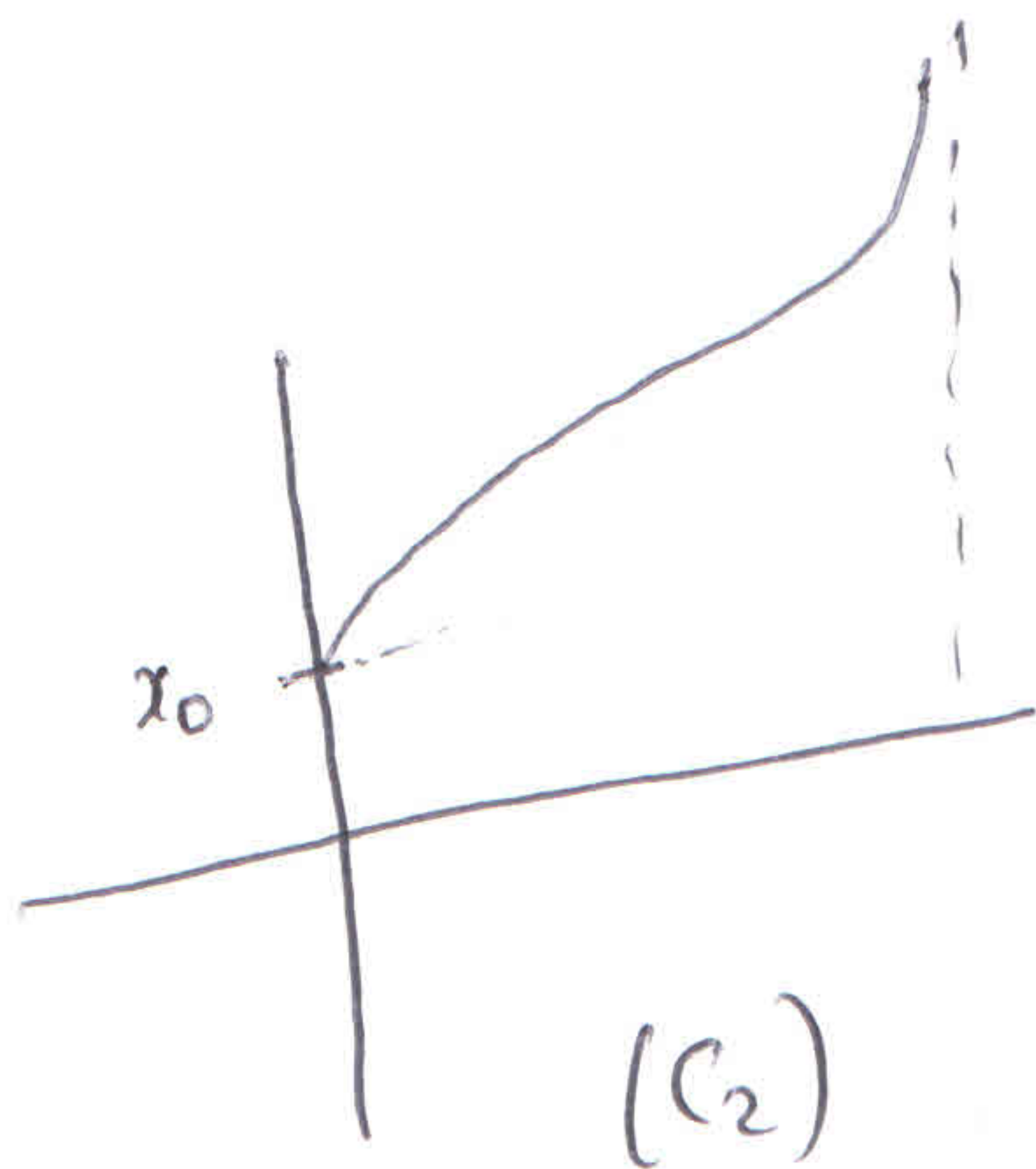
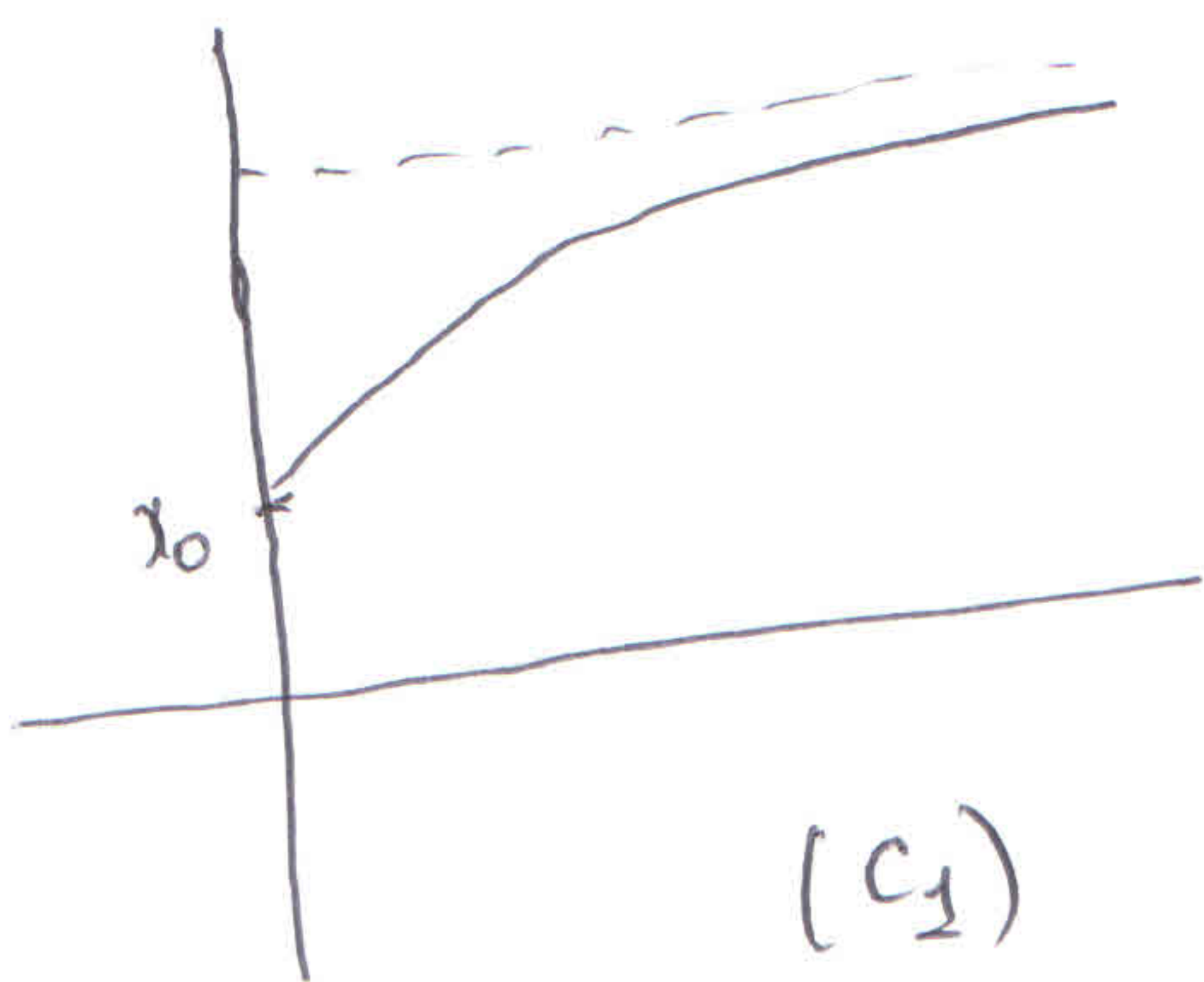
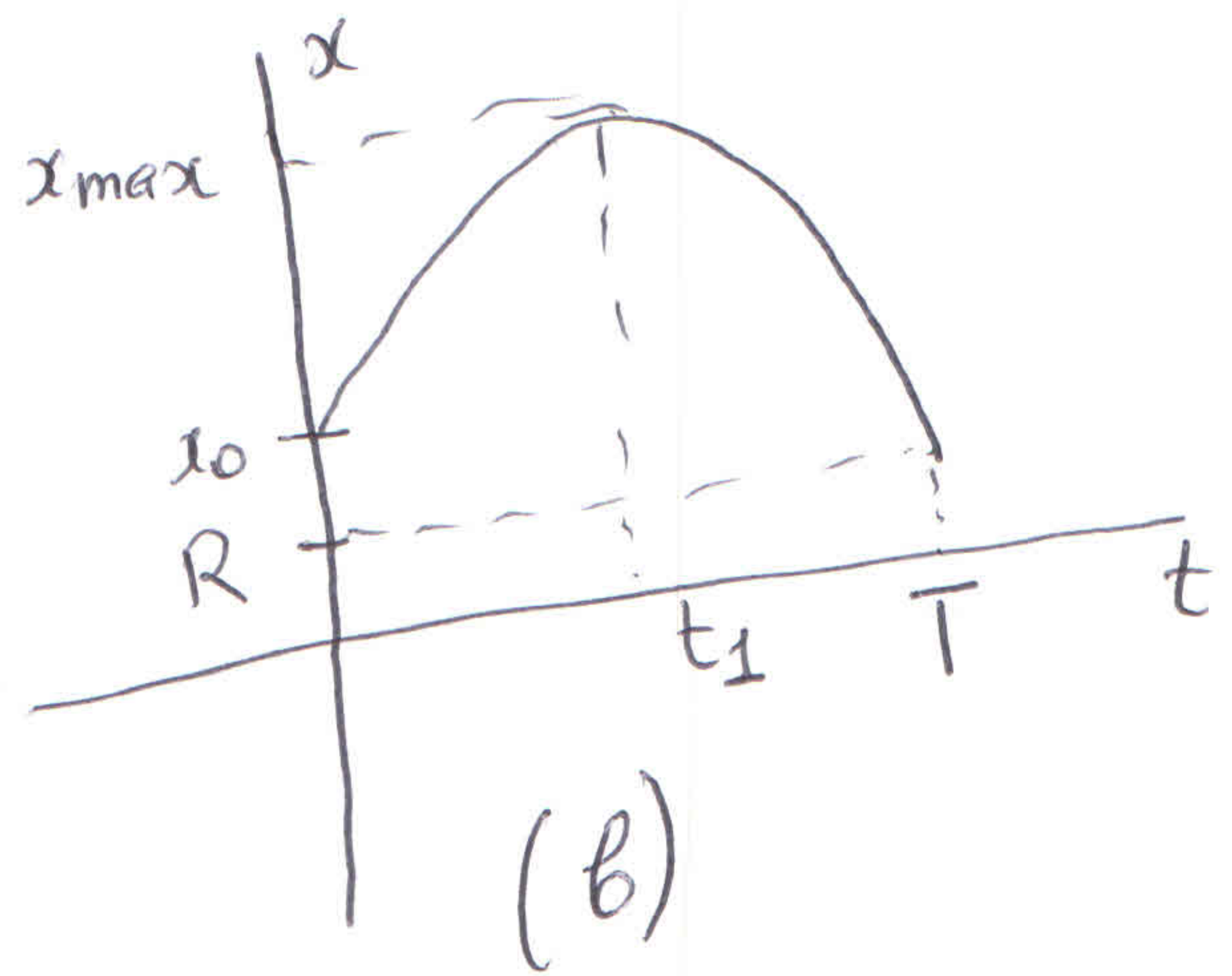
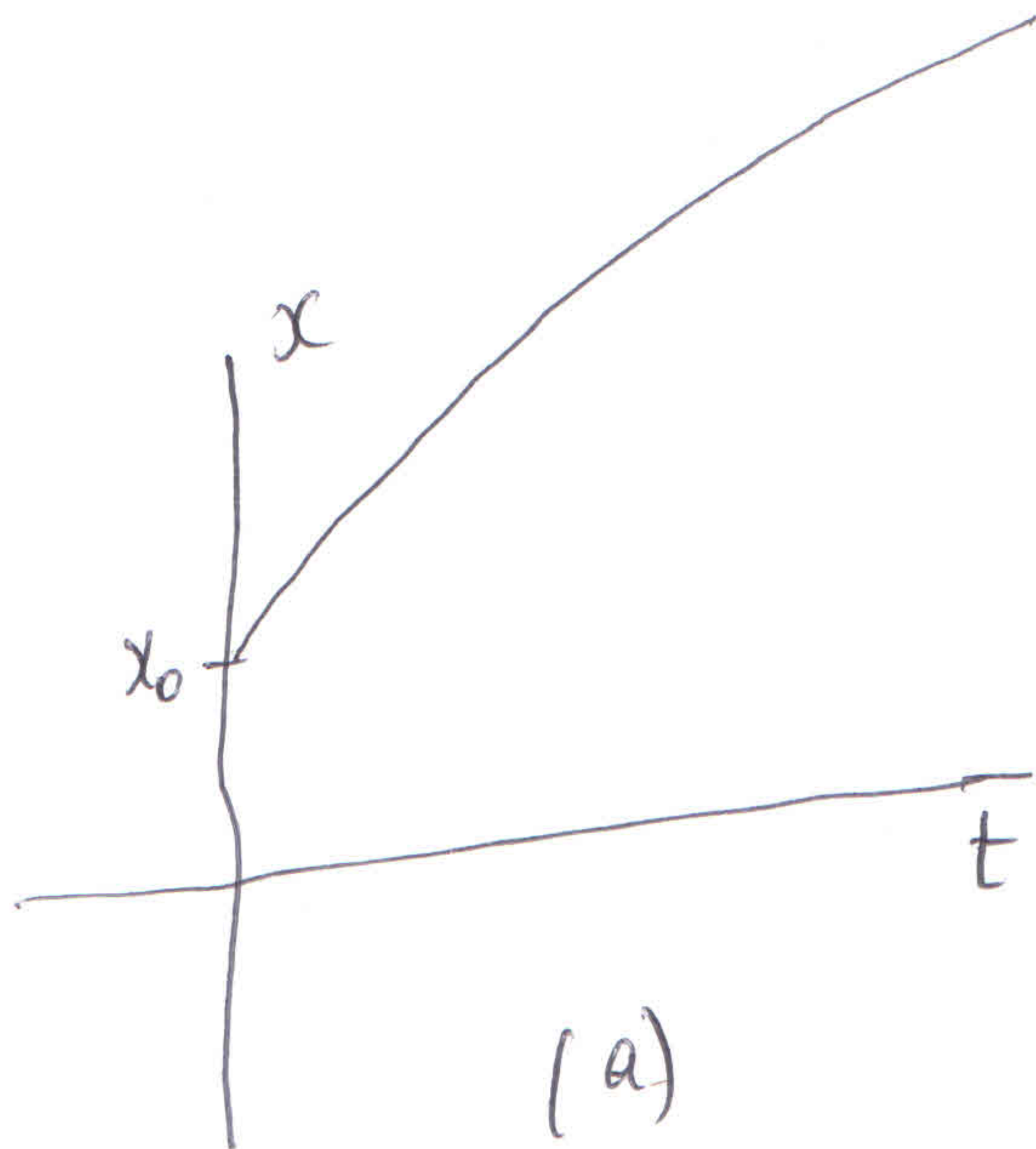
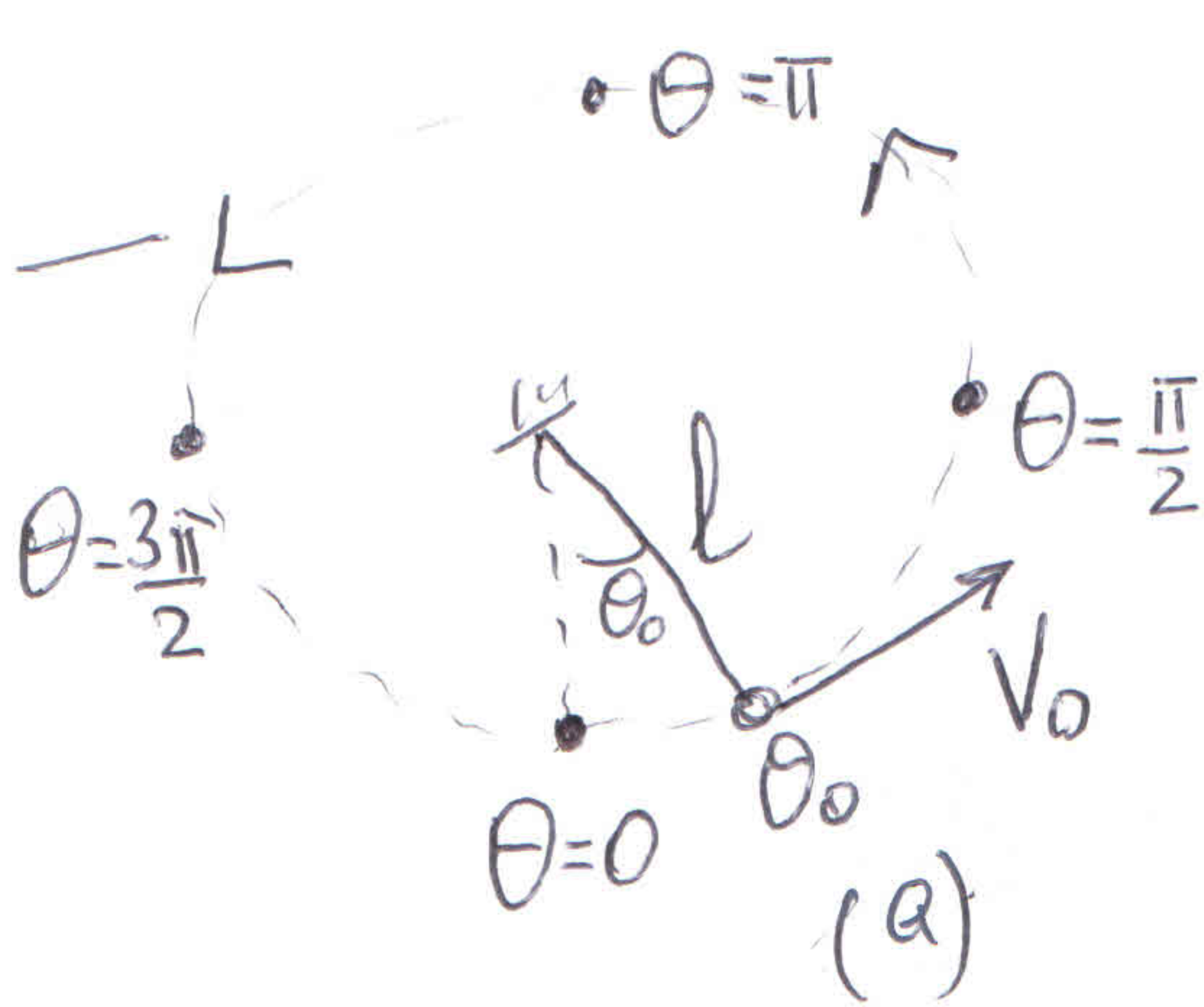
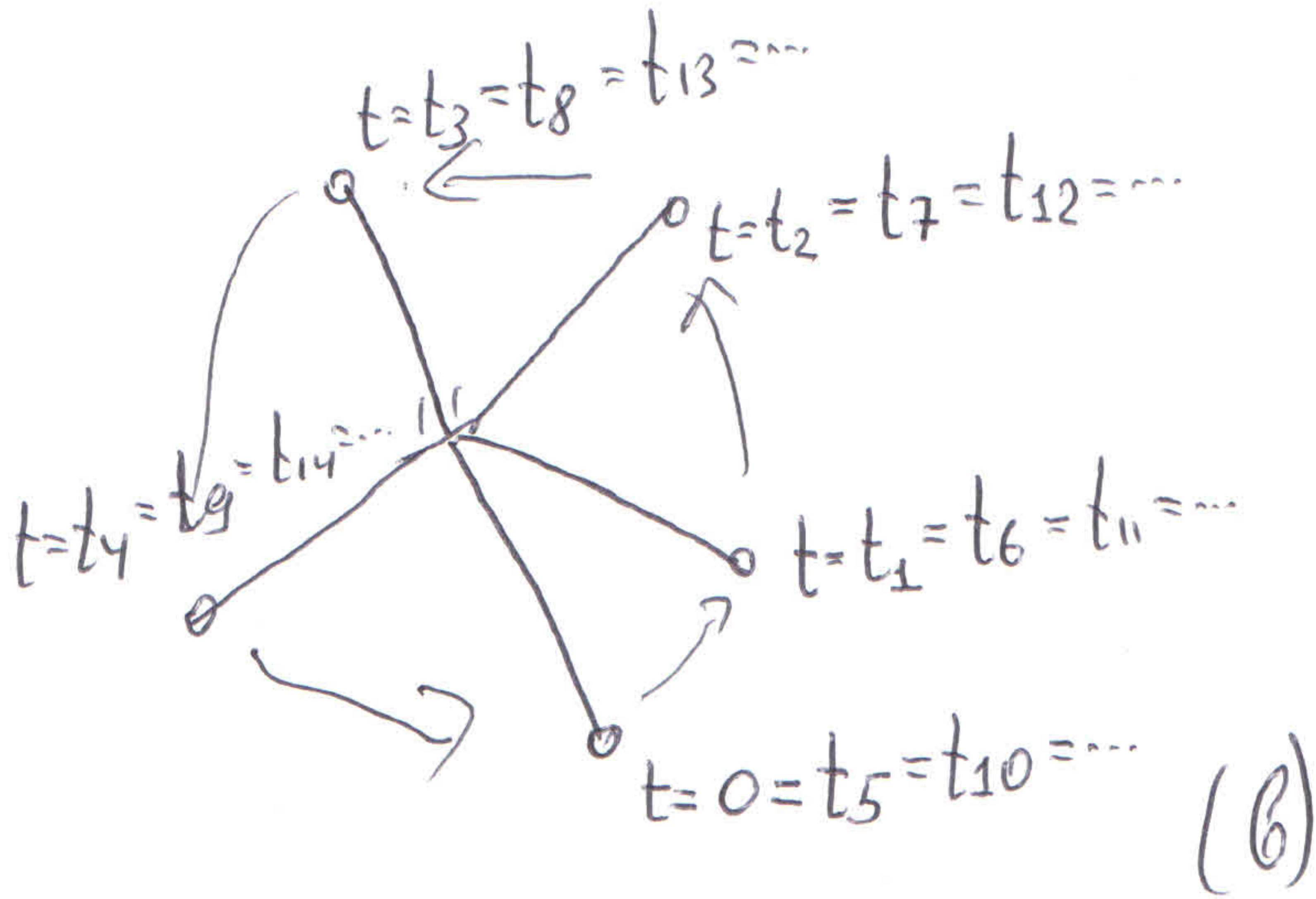


fig. 4.7. Solution of the two body problem: either (a) or (b); (c₁), (c₂), (c₃): impossible

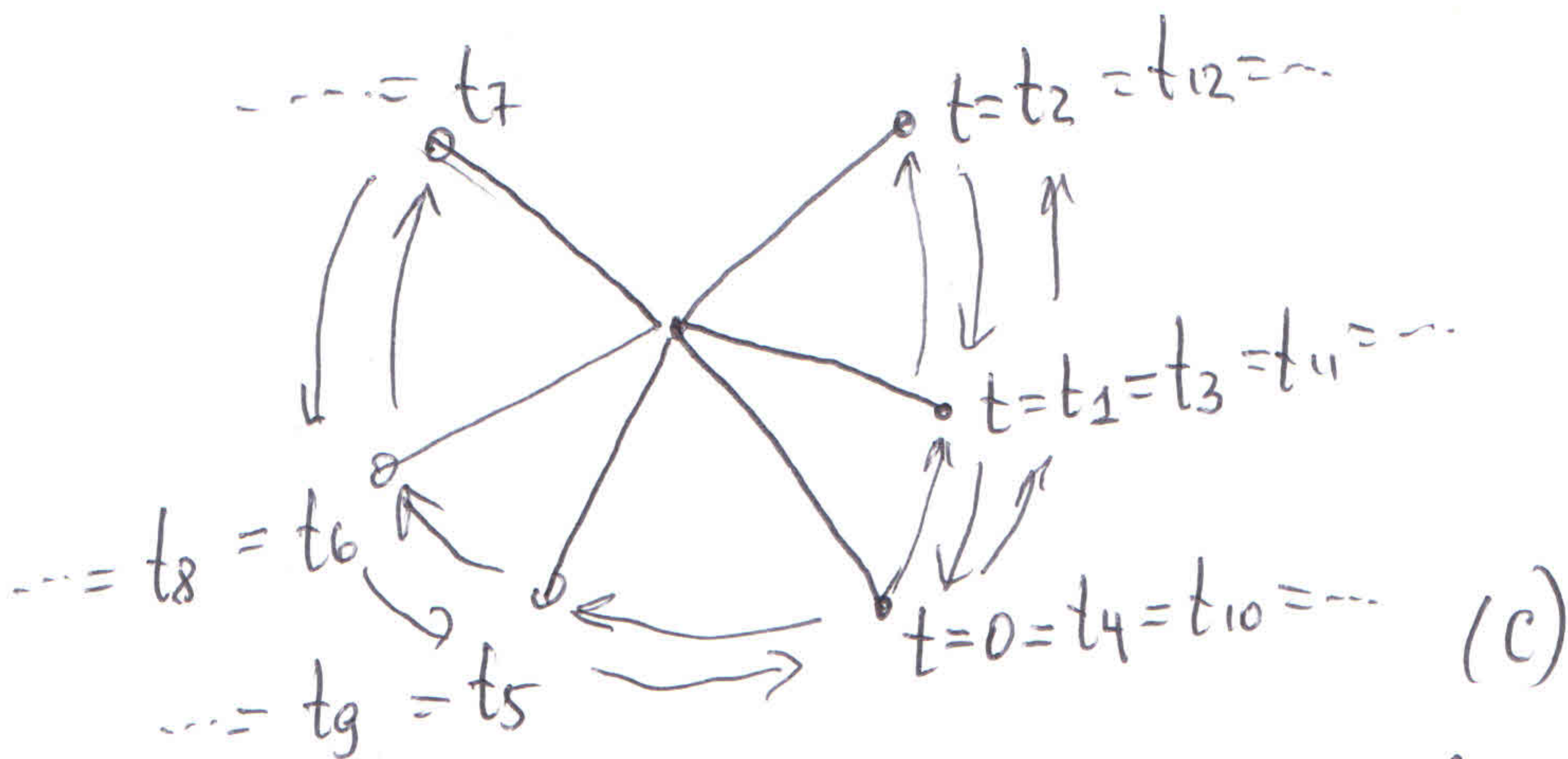
positive direction
(anticlockwise)



θ : in rad
defined
 $\text{mod } 2k\pi, k \in \mathbb{Z}$



Pendulum turns over.
Position of the
pendulum at
time moments
 $t=0 < t_1 < t_2 < t_3 < \dots$
It is so $\Leftrightarrow V_0 > V_{0, \text{crit}}$



Pendulum
oscillates.
Position of the
pendulum at
time-moments
 $t=0 < t_1 < t_2 < t_3 < \dots$
It is so $\Leftrightarrow V_0 < V_{0, \text{crit}}$

fig. 4.8. Pendulum or swing.

- (a):: All possible positions: circle S^1 , with coordinate θ in rad; $\theta=0$: stable equilibrium, $\theta=\pi$: unstable equilibrium.
 (b):: the case that the pendulum turns over
 (c):: the case that the pendulum oscillates.

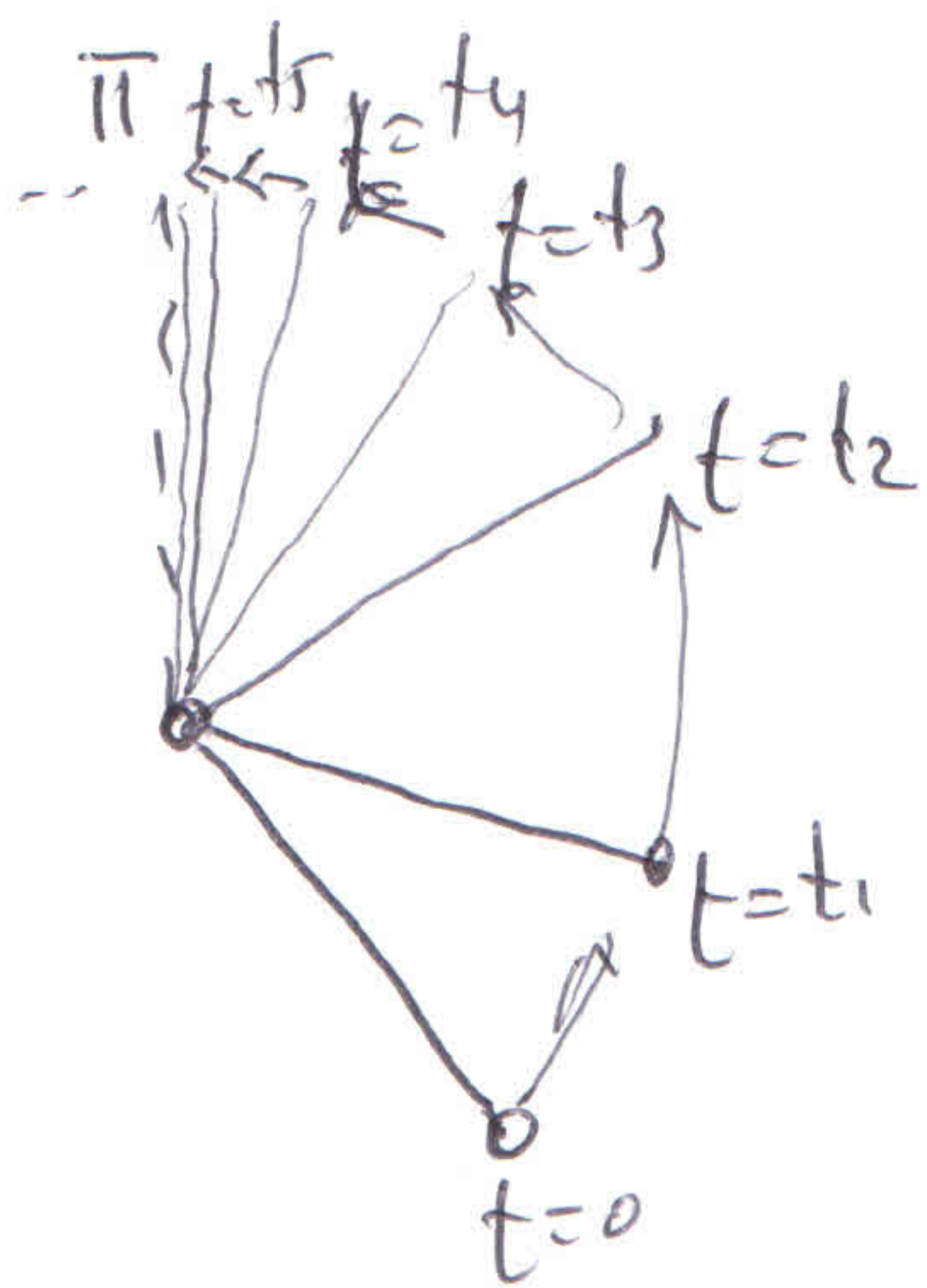


fig. 4.9. If $V_0 = V_{0, \text{crit}}$
 the pendulum will approach $\theta = \pi$
 (position of unstable equilibrium),
 but will never be in this position.



$V_0 > V_{0, \text{crit}}$



$V_0 < V_{0, \text{crit}}$



$V_0 = V_{0, \text{crit}}$

fig. 4.10. The image of the solution:
 $\{ \theta(t), t > 0 \}$

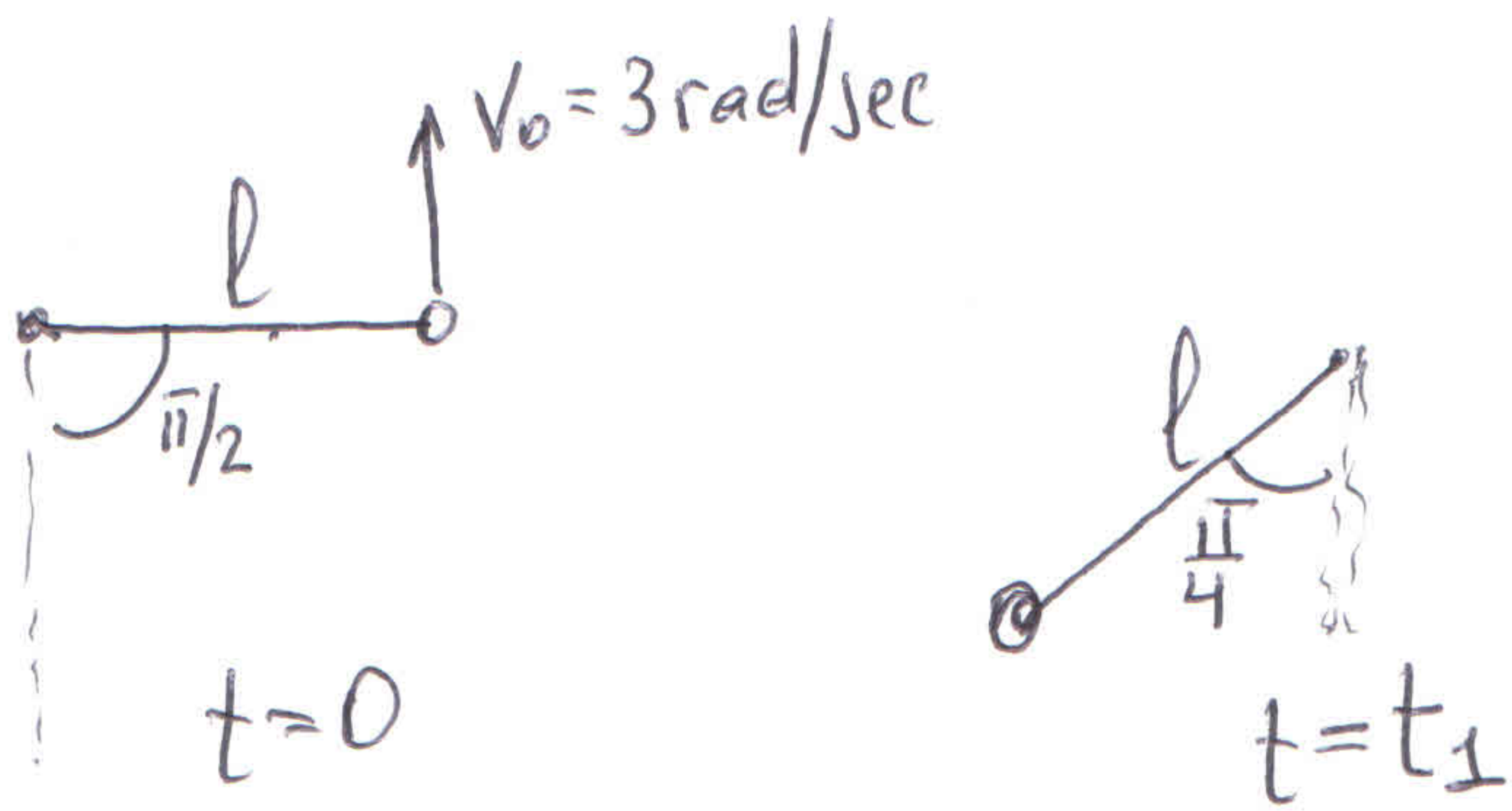


fig. 4.11. Problem in section 4.6.3: to find t_1 .

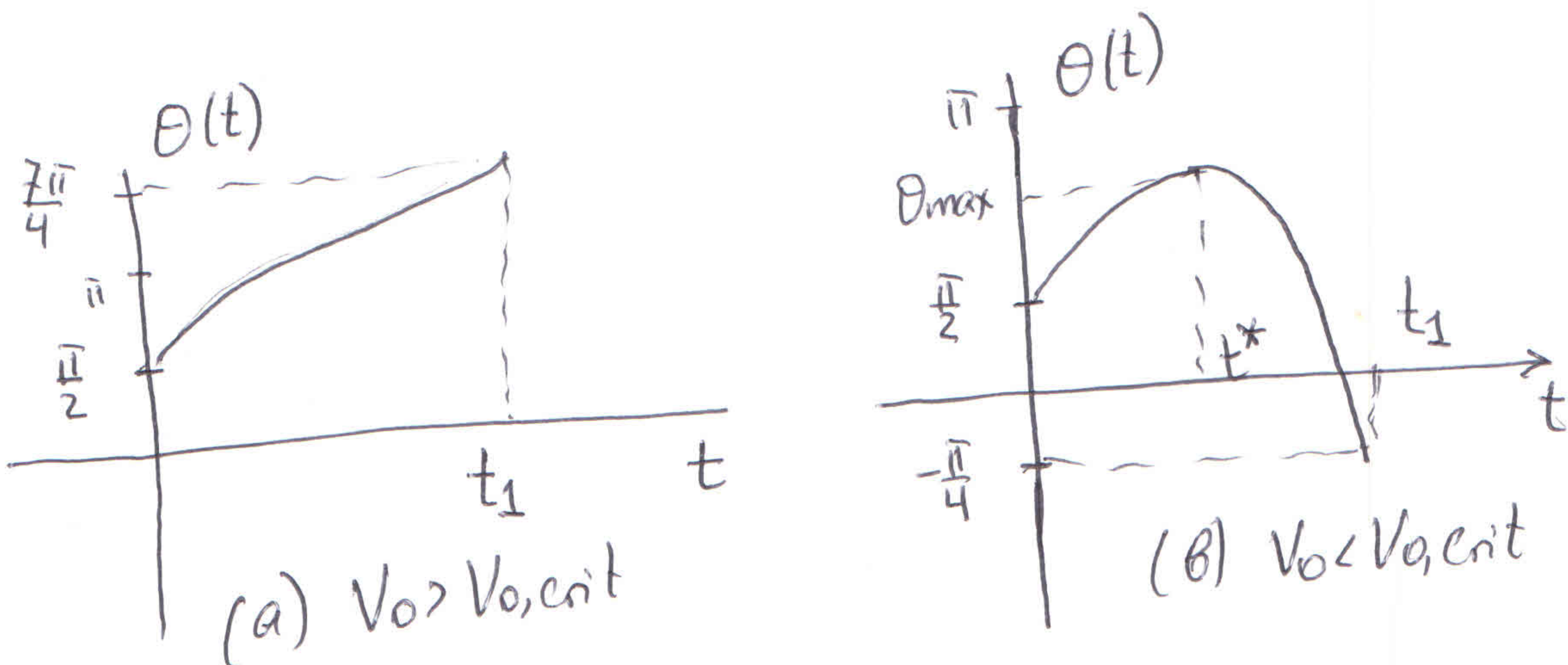


fig. 4.12. $\theta(t)$ as a function $\mathbb{R} \rightarrow \mathbb{R}$ if $v_0 > v_{0,crit}$ and $v_0 < v_{0,crit}$

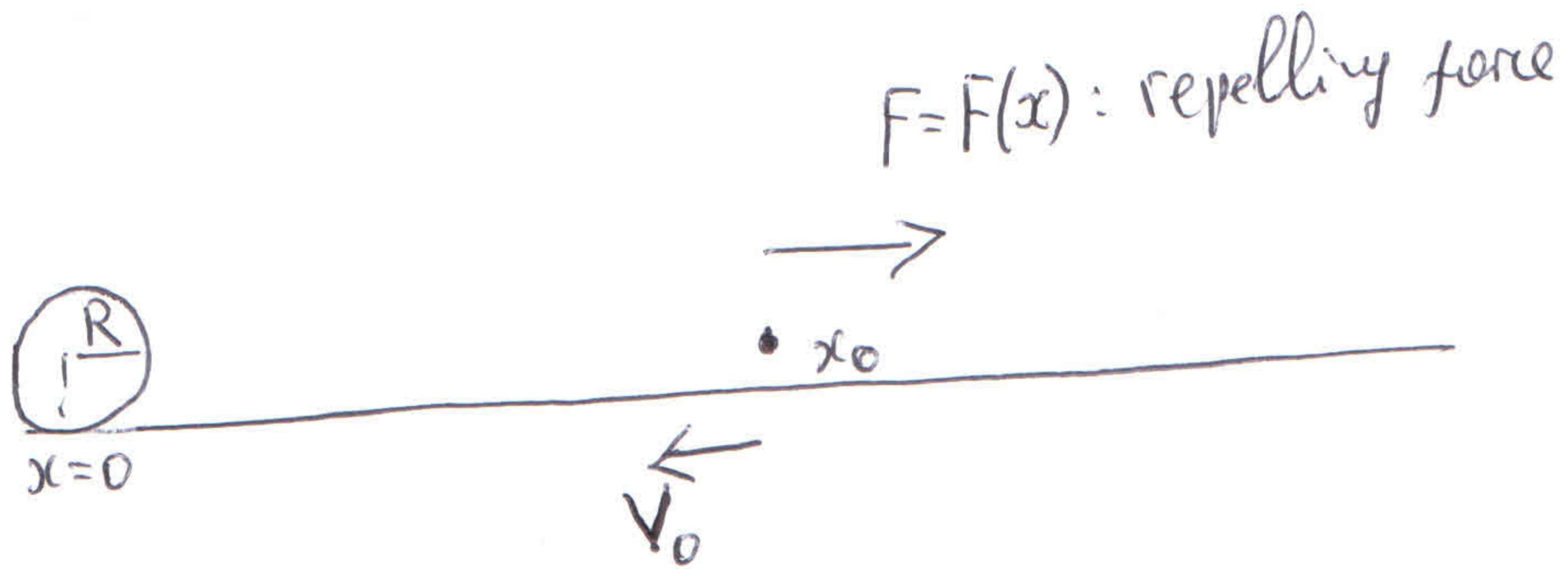


fig. 4.13. Another version of two problem (exercise 5).

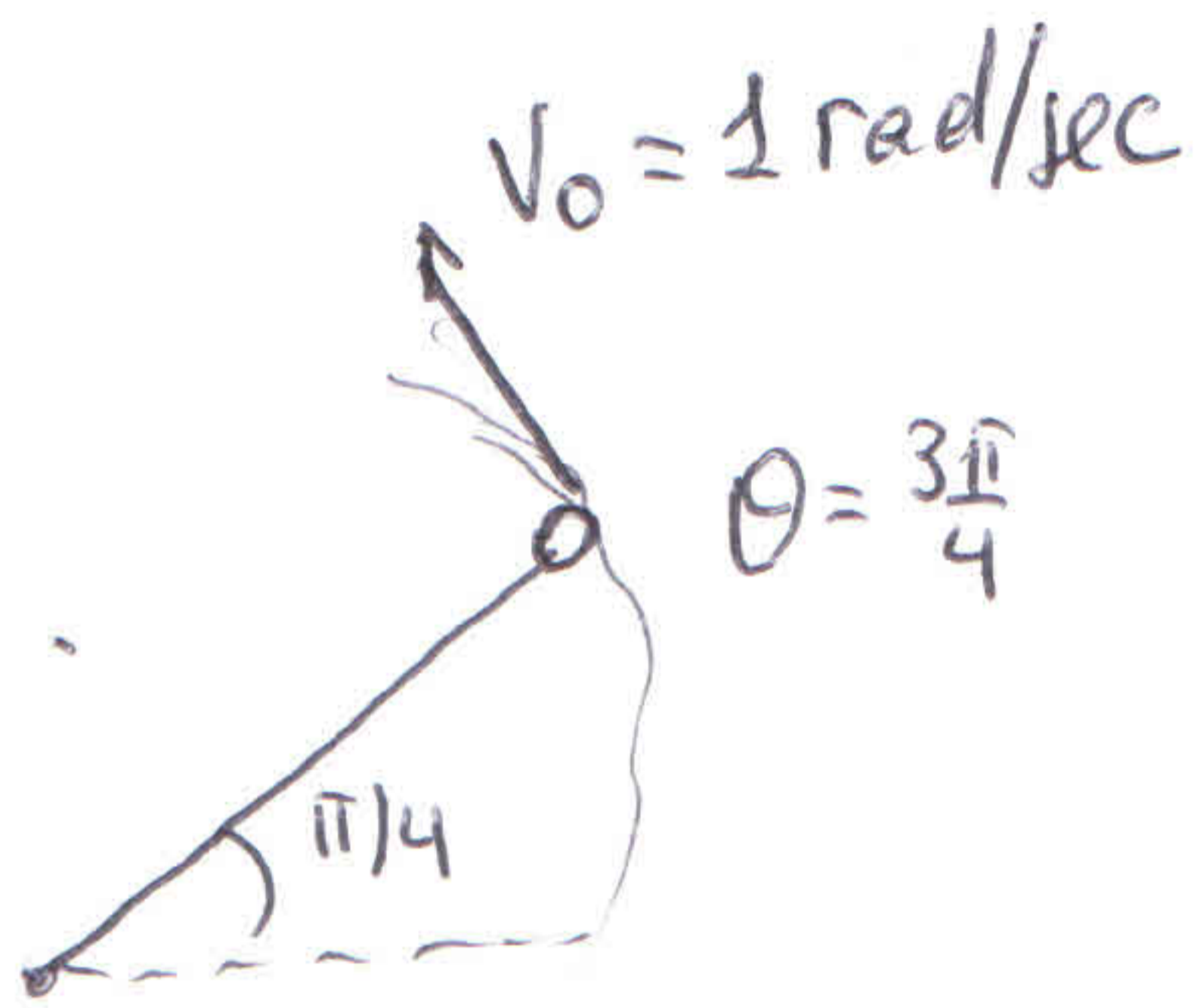


fig. 4.14. For exercise 8.