ODEs - 104285. Semester: Spring. Year: 2012

HW-1. Deadline: Monday, April 2, 6 pm

1. . Let $f(x) = (x-1)^2(x-2)^3(x+1)^4(x+2)^5$ and let $x_1(t), ..., x_7(t)$ be the solutions of the equation $x'(t) = f(x(t))$ satisfying the initial condition
$x_1(0) = -1.5, \quad x_2(1) = -1, \quad x_3(2) = -0.5,$
$x_4(3) = 0.5, \; x_5(4) = 1, \quad x_6(5) = 1.5, \quad x_7(6) = 1.5$
and defined on maximal possible interval (t^-, t^+). Find, for each of these solutions, t^- and t^+ and draw the 7 graphs, of $x_1(t), ..., x_7(t)$, in the same (t, x) plane. Do not use calculator.

2. Let $x_1(t), ..., x_7(t)$ be the solutions of the equation $x'(t) = sin(e^{x(t)})$ satisfying the initial condition
$x_1(0) = 2, \quad x_2(1) = 2, \quad x_3(0) = 3, \quad x_4(1) = 3,$
$x_5(-1) = 4, \quad x_6(-1) = 5, \quad x_7(-1) = 6$
and defined on maximal possible interval (t^-, t^+). Find, for each of these solutions, t^- and t^+ and draw the 7 graphs, of $x_1(t), ..., x_7(t)$ in the same (t, x) plane. For each of the functions $x_1(t), ..., x_7(t)$ find its value at the inflection point (nikudat pitul). Probably you will need a calculator.

3. Let $x(t)$ be solution of the equation $x'(t) = f(x(t))$, where $f(x)$ is a function given below, satisfying the initial condition $x(3) = 5$ and defined on maximal possible interval (t^-, t^+). Find t^-, t^+, draw the graph of $x(t)$ and answer the following question: for which $a \in \mathbb{R}$ there exists $t_1 \in (t^-, t^+)$ such that $x(t_1) = a$? For all such a give a formula for t_1 (integral in the formula is OK).

a) $f(x) = (x^2 - 1)sinx$ b) $f(x) = sinx + cosx$

4. Let $x(t)$ be the solution of the equation $x'(t) = f(x(t))$, where $f(x)$ is the functions given below, satisfying the initial condition $x(3) = 5$ and defined on the maximal possible interval (t^-, t^+). Find t^-, t^+, find $x(3.5)$, and draw the graph of $x(t)$.

a) $f(x) = x^2 + 30$ (no integrals in the answers)

b) $f(x) = x^2 - 30$ (no integrals in the answers)

c) $f(x) = \frac{x^7 - 100}{x^6 + 1}$
(integrals are OK only if they are convergent (metkansim))

d) $f(x) = \frac{x^7 - 10^6}{x^4 + 1}$
(integrals are OK only if they are convergent (metkansim))