ODEs - 104285. Semester: Spring. Year: 2011

HW-2. Deadline: Monday, March 21, 6 pm

Dictionary:

inflection point = nikudat pitul' = (Rus) tochka peregiba
convex = kamur = (Rus) vipuklii
concave = kaur = (Rus) vognutii

Remind that a point $a \in \mathbb{R}$ is called an inflection point of a function $f(x)$ if for some positive ϵ the function $f(x)$ is concave in the interval $(a-\epsilon, a)$ and convex in the interval $(a, a+\epsilon)$, or vise a versa.

1. Let $f(x) \in C^1(\mathbb{R})$, $f(a) = f(b) = 0$, and $f(x) \neq 0$ for $x \in (a, b)$.
Let $t_0 \in \mathbb{R}, x_0 \in (a, b)$. Let $x(t)$ be the solution of the equation $x'(t) = f(x(t))$ satisfying the initial condition $x(t_0) = x_0$ and defined for all $t \in \mathbb{R}$.
Prove that the function $x(t)$ has at least one inflection point.

2. Draw the graph of a function $f(x) \in C^1(\mathbb{R})$ such that
$f(0) = f(100) = 0, f(x) \neq 0$ for $x \in (0, 100)$ and such that for any $t_0 \in \mathbb{R}$ and any $x_0 \in (0, 200)$ the solution of the equation $x'(t) = f(x(t))$ satisfying the initial condition $x(t_0) = x_0$ and defined for all $t \in \mathbb{R}$ has 10 inflection points. Prove that for your graph of $f(x)$ it is so.

3. Let $x_1(t), ..., x_7(t)$ be the solutions of the equation $x'(t) = sin(e^{x(t)})$ satisfying the initial condition 3.1, ..., 3.7 below and defined for all t. Draw the 7 graphs, of $x_1(t), ..., x_7(t)$ in the same (t, x) plane. For each of the functions $x_1(t), ..., x_7(t)$ find its value at the inflection point.

1.1. $x(0) = 2$ 1.2. $x(1) = 2$ 1.3. $x(0) = 3$ 1.4. $x(1) = 3$

1.5. $x(-1) = 4$ 1.6. $x(-1) = 5$ 1.7. $x(-1) = 6$

4. Let $f(x) = (x-1)^2(x-2)^3(x+1)^4(x+2)^5$ and let $x_1(t), ..., x_7(t)$ be the solutions of the equation $x'(t) = f(x(t))$ satisfying the initial condition 4.1, ..., 4.7 below and defined for all t. Draw the 7 graphs, of $x_1(t), ..., x_7(t)$ in the same (t, x) plane.

4.1. $x(0) = -1.5$ 4.2. $x(1) = -1$ 4.3. $x(2) = -0.5$ 4.4. $x(3) = 0.5$

4.5. $x(4) = 1$ 4.6. $x(5) = 1.5$ 4.7. $x(6) = 1.5$