ODEs - 104285. Semester: Spring. Year: 2012

HW-3. Deadline: Monday, April 30, 6 pm

1. For each of the given equations and for each of the initial conditions, draw the graph of the solution $x(t)$ defined on maximal possible time-interval (t^-, t^+), find t^- and t^+, find the points of local maxima and local minima, and find the limits of $x(t)$ as $t \to t^+$ and $t \to t^-$:

1.1 $x'(t) = (x^2(t) - 7x(t) + 10) \cdot (t + 3)$

a) $x(0) = 6$ b) $x(0) = 3$ c) $x(0) = 1$

1.2 $x'(t) = (x^2(t) - 7x(t) + 10) \cdot \frac{t+1}{t^2+1}$

a) $x(0) = 6$ b) $x(0) = 3$ c) $x(0) = 1$

1.3 $x'(t) = (x^2(t) - 7x(t) + 10) \cdot \frac{t^2}{(t^3+1)^2}$

a) $x(0) = 6$ b) $x(0) = 3$ c) $x(0) = 1$

2. Let $x(t)$ be solution of the equation $x'(t) = (-2x(t) + t + 5)^2$ satisfying the initial conditions

a) $x(0) = 5/2$ b) $x(0) = 2$ c) $x(0) = 3$

and defined on maximal possible time interval (t^-, t^+). Find t^-, t^+ and a formula for $x(t)$ (without integrals). If $t^+ = \infty$ find $\lim_{t\to\infty} \frac{x(t)}{t}$.

3. Find a formula, with an integral, for the solution $x(t)$ of the equation $x'(t) = -x(t) + \frac{\sqrt{t^2+1}}{x^2(t)}$ satisfying the initial condition $x(0) = 1$.

4. Find a formula, without integrals, for the solution of the equation

$$x'(t) = \frac{x+1}{t} + 1$$

satisfying the initial condition $x(1) = 0$

(Hint: make substitution $\widetilde{x}(t) = x(t) + 1$).