ODEs - 104285. Semester: Spring. Year: 2011

HW-3. Deadline: Monday, April 4, 6 pm

1. The level of love $x(t)$ between Sara and Itsik (who listened my course and know optimization by feedback rather than by rigid plan) changes according to the equation $x'(t) = f(x(t))$, where $f(x) = x(1-x) - \frac{1}{2}x$. Here t is measured in weeks and $x(t)$ is a part of the love of Romeo and Djulyeta.

In how much time the level of love will be in the interval $(0.49, 0.51)$ if at the initial time $t = 0$ the level of love is

1.1. $x(0) = 0.1$ 1.2. $x(0) = 0.9$

2. Let $x(t)$ be solution of the equation $x'(t) = f(x(t))$, where $f(x)$ is a function given below, satisfying the initial condition $x(3) = 5$. For which $a \in \mathbb{R}$ there exists t_1 such that $x(t_1) = a$? For all such a give a formula for t_1 (integral in the formula is OK).

2.1. $f(x) = (x^2 - 1)sinx$ 2.2. $f(x) = sinx + cosx$ 2.3. $f(x) = ln(x^2 + 2x + 1.5)$

3. For each of the function given in problem 2, draw the phase portraits for the equation $x'(t) = f(x(t))$ and distinguish all stable singular points.

4. For each of the function given in problem 2, draw in the same (t, x)-plane four graphs - the graphs of solutions of the equation $x'(t) = f(x(t))$ satisfying the initial condition $x(0) = -1,\ x(0) = 1/2,\ x(0) = 3,\ x(0) = 4$.

5. Let $x(t)$ be the solution of the equation $x'(t) = f(x(t))$, where $f(x)$ is one of the functions given below, satisfying the initial condition $x(3) = 5$. Find $x(10)$ (a number, no integrals).

5.1. $f(x) = x^2 + 4$ 5.2. $f(x) = x^2 - 4x + 3$ 5.3.$f(x) = \sqrt{x+5}$

6. Find the formulas for two different solutions of the equation $x'(t) = x^2(t)$ such that each of them is defined on maximal possible time-interval, one of them goes to ∞ as $t \to t^+$ and the other goes to 0 as $t \to t^+$.

7. Find the maximal time-interval and the formula for the solution of the equation

$$7.1\ \ x'(t) = \frac{\sqrt{x^2(t) - 1}}{x(t)}, \qquad 7.2.\ \ x'(t) = \frac{\sqrt{x^2(t) + 1}}{x(t)}$$

satisfying the initial condition $x(0) = -2$. Draw the graphs of this solutions.