ODEs - 104285. Semester: Spring. Year: 2011

HW-4. Deadline: Monday, April 11, 6 pm

1.(*) A rocket has been launched up from the Earth surface with the initial velocity $v_0 = rv_{0,crit}$, where $v_{0,crit} = \sqrt{2gR}$ is the "escape velocity" and $0 < r < 1$. Here R is the radius of the Earth. Let $t_1 = t_1(r)$ be the time in which the rocket will reach its maximal height. Obtain explicit (without integrals) formula for the function $t_1(r)$ as $0 < r < 1$. Is it true that $t_1(r) \to \infty$ as $r \to 1$? Is it true that $t_1(r)$ is an increasing function? Find the initial velocity v_0 (approximately) such that the rocket will reach its maximal height in two weeks after it has been launched. Solving the problem use the equation $x'' = -\frac{gR^2}{x^2}$ where $x = x(t)$ is the distance between the rocket and the center of the Earth.

2.(*) A fight between two armies (many years ago) is described by equations $x_1' = -kw_2x_2,\ x_2' = -kw_1x_1$ where $x_1 = x_1(t)$ and $x_2 = x_2(t)$ are the number of soldiers in the armies, w_1 and w_2 is the amount of weapon in the armies (assumed to be constant during the fight) and k is a certain fixed positive number. At the beginning of the fight the armies had $x_1(0) = s_1$ and $x_2(0) = s_2$ soldiers. In which time t_1 the fight will be over, i.e. one of the armies will have no soldiers ($x_1(t_1) = 0$ or $x_2(t_1) = 0$)? Give an explicit formula (without integrals) for t_1 as a function of k, w_1, w_2, s_1, s_2. After that fix $k = 1,\ w_2 = 40,\ w_1 = 20,\ s_2 = 1000$ so that $t_1 = t_1(s_1)$ and draw the graph of the function $t_1(s_1)$ as $0 < s_1 < \infty$.

Help. As I showed in the lecture the first army wins the fight if and only if $s_1 > s_2\sqrt{w_2/w_1}$, therefore you should consider two cases: 1) $s_1 > s_2\sqrt{w_2/w_1}$ and 2) $s_1 < s_2\sqrt{w_2/w_1}$. In the first case t_1 is the time such that $x_2(t_1) = 0$ which can be found from the equation $x_2'' = k^2w_1w_2x_2$ following from the given system of equations. In the second case t_1 is the time such that $x_1(t_1) = 0$ which can be found from the equation $x_1'' = k^2w_1w_2x_1$ also following from the given system of equations. It is also worth to think if the two formulas (for the first and the second case) can be joined into one formula, and what happens with t_1 if $s_1 = s_2\sqrt{w_2/w_1}$

3. A big body which does not move repels a small body of mass 3 kg with the force $1/x$ (in $kg \cdot m/sec^2$) where $x = x(t)$ is the distance between the bodies (in meters). At the initial time-moment the distance between the bodies is 4 meters and the small body has velocity 2 m/sec towards the big body. Find the minimal distance x_{min} between the bodies (without integrals in the answer) and find time t_1 such that $x(t_1) = x_{min}$ (integrals in the answer OK).