

ODEs - 104285. Semester: Spring. Year: 2011

HW-5. Deadline: Sunday, May 15, 2 pm

1. A big body, which does not move, attracts a small body of mass m kg with the force

$$a) F = \frac{1}{x \ln x}, \quad b) F = \frac{1}{\sqrt{x}} \quad c) F = \frac{1}{x\sqrt{x}}$$

$kg \cdot m/sec^2$, where x is the distance between the bodies (in meters). At the initial time ($t = 0$) the distance between the bodies is 10 meters and the initial velocity of the small body is v_0 m/sec directed in such a way that the distance between the bodies starts to increase. In which time t^* the distance between the bodies will be 20 meters? The answer depends on v_0 and m . If the answer is not unique you should find all answers. Integrals in the answers are OK.

2. A pendulum is described by the equation

$$\theta'' = -\frac{g}{l} \sin \theta$$

where l is the pendulum length, the angle θ is measured in radians, the positive direction is anticlockwise, and $\theta = 0 \pmod{2k\pi}$ corresponds to the position of stable equilibrium. At the initial time $t = 0$ the pendulum is horizontal ($\theta(0) = \theta_0 = \pi/2$) and has initial velocity $\theta'(0) = v_0 = \sqrt{g}$ rad/sec directed anticlockwise. In which time t_1 the pendulum will pass (for the first time) the position of stable equilibrium? This means that

$$t_1 = \min\{t \geq 0 : \theta(t_1) = 0 \pmod{2\pi k}\}$$

The answer depends on the length l and you should solve the problem for

a) the length l is 1 meter b) the length l is 3 meters.

Integrals in the answers are OK. Which velocity the pendulum will have at the time t_1 ?