ODEs - 104285. Semester: Spring. Year: 2011

HW-6. Deadline: Monday, May 30, 2 pm

1. Find the solution $x(t)$ of the equation given below satisfying the initial condition

$$x(0) = 0.$$

Without integrals in the answer. Try to understand the maximal possible time interval for the solution. Draw the graph of $x(t)$ containing as much information as you can determine.

$$1.1.\quad x' = \frac{x}{t+1} + t^2$$

$$1.2.\quad x' = (x+2)(x-3)\cdot sin(t-1)$$

$$1.3.\quad x' = \frac{x(t+1)}{x^2+(t+1)^2} \qquad\qquad \text{hint: make a change } \widetilde{t} = t+1$$

$$1.4.\quad x' = \frac{x-t}{3t-2x+1}$$

2. Let

$$x' = Ax,\ A = \begin{pmatrix} 3 & 1 & 4 & 0 \\ -2 & 3 & 2 & 1 \\ 7 & 4 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{pmatrix},\quad x = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{pmatrix},\quad x(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}.$$

Find $a, b, c \in \mathbb{R}$ such that $x_2(t) = a + bt + ct^2 + o(t^2)$ as $t \to 0$.

3. Give an example of a basis of the vector space of all solutions of the system

$$x_1' = 5x_1 - x_2,\quad x_2' = 18x_1 - 6x_2$$

and find an explicit formula for the solution of this system satisfying the initial conditions $x_1(0) = 0,\ x_2(0) = 1$.

4. Give an example of a basis of the vector space of all solutions of the system

$$x' = TJT^{-1}x,\quad T = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix},\quad J = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix}$$

and find the solution satisfying the initial condition $x_1(0) = 1,\ x_2(0) = x_3(0) = 0$. (Hint: make a linear change $x = Ty$).