ODEs - 104285. Semester: Spring. Year: 2012

HW-4. Deadline: Monday, May 21, 6 pm

1. A rocket has been launched up from the Earth surface with the initial velocity $v_0 = (1-\epsilon)v_{0,crit}$, where $v_{0,crit} = \sqrt{2gR}$ is the "escape velocity" (or second cosmic velocity), $\epsilon > 0$, and R is the radius of the Earth. Let $t^* = t^*(\epsilon)$ be the time in which the rocket will reach its maximal height. Obtain explicit (without integrals) formula for the function $t^*(\epsilon)$ and characterize the behavior of $t^*(\epsilon)$ ar $\epsilon \to 0$. For which ϵ (approximately) the rocket will reach its maximal height in a month after it has been launched?

Solving the problem use the equation $x'' = -\frac{gR^2}{x^2}$ where $x = x(t)$ is the distance between the rocket and the center of the Earth. You will have to compute the integral of the form $\int \frac{dx}{\sqrt{\frac{a}{x}+b}}$. Enjoy integration techniques from hedva or infi.

2. A big body which does not move repels a small body of mass 3 kg with the force $1/x$ (in $kg \cdot m/sec^2$) where $x = x(t)$ is the distance between the bodies (in meters). At the initial time-moment the distance between the bodies is 4 meters and the small body has velocity 2 m/sec towards the big body. Find the minimal distance x_{min} between the bodies (without integrals in the answer) and find time t_1 such that $x(t_1) = x_{min}$ (integrals in the answer OK).