ODEs - 104285. Semester: Spring. Year: 2012

HW-6. You should do it by June 18

1. Let $P(\lambda) = (\lambda - \lambda_0)^r$ where r is a positive integer. Prove that the equation $P(\frac{d}{dt})x(t) = 0$ has solutions $x^i e^{\lambda_0 t},\ i = 0, 1, ..., r-1$ and prove that these solutions are linearly independent.

No complex numbers in the final answers to problems 2-5

2. Let $x(t) = e^{2t}cos(5t)$.

a. Find $P(\frac{d}{dt})(x(t))$ if $P(\lambda) = \lambda^3 - 2\lambda^2 + \lambda - 1$

b. Let $P(\lambda) = Q(\lambda)(\lambda^2 - 4\lambda + a)$.
Find $a \in \mathbb{R}$ such that $P(\frac{d}{dt})(x(t)) = 0$ for **any** polynomial $Q(\lambda)$.

3. Find the set of all solutions of the following equations:

3.1. $x''''''''''(t) = x(t)$ (the 10-th derivative of $x(t)$ is equal to $x(t)$)

3.2. $x'''''''(t) + x(t) = 0$ (the 7-th derivative of $x(t)$ plus $x(t)$ is 0).

3.3. $x'''(t) + x'(t) = 0$

4. Find the set of all solutions of the equation $P(\frac{d}{dt})(x(t)) = 0$ where

4.1. $P(\lambda) = (\lambda^2 + 1)^4$

4.2. $P(\lambda) = (\lambda^2 - 1)^4$

4.3 $P(\lambda) = (\lambda^2 - 1)^4$

4.4. $P(\lambda) = (\lambda^2 + 1)^4(\lambda + 1)$

4.5. $P(\lambda) = (\lambda^2 - 2\lambda + 2)^7(\lambda^2 - 4\lambda + 13)^9$

5. Find a partial solution of the following equations using the method of variation of constant:

5.1. $x'' + x = ln\, t,\quad x = x(t),\ t > 0$

5.2. $x''' + x = \sqrt{t},\quad x = x(t),\ t > 0$