ODEs - 104285. Semester: Spring. Year: 2012

HW-7. You should do it by Sunday June 24 evening

1. Find a partial solution of the equation

$$P(\frac{d}{dt})(y(t)) = e^t sint + e^{2t}cos(3t) + cos(3t) + e^{4t},$$

where

$P(\lambda) = (\lambda^2 - 2\lambda + 2)^7(\lambda^2 - 4\lambda + 13)^9$
(requires 8-10 min)

2. Let

$$x' = Ax,\ A = \begin{pmatrix} 3 & 1 & 4 & 0 \\ -2 & 3 & 2 & 1 \\ 7 & 4 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{pmatrix},\quad x = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{pmatrix},\quad x(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}.$$

Find $a, b, c \in \mathbb{R}$ such that $x_2(t) = a + bt + ct^2 + o(t^2)$ as $t \to 0$.

(requires 5-7 min)

3. Find the set of all solutions of the system $x_1' = x_1 + x_2,\ x_2' = 6x_1 - 4x_2$ (without exponent of a matrix)

(requires 5-8 min)

4. Find e^A where $A = \begin{pmatrix} 1 & 1 \\ 6 & -4 \end{pmatrix}$ (without series in the answer).

Hint: find solution $x(t)$ of the system in problem 3 satisfying the initial condition $x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. you know that $x(t) = e^{tA}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and consequently $e^A\begin{pmatrix} 1 \\ 0 \end{pmatrix} = x(1)$ which allows you to find the first column of e^A. In a similar way you can find the second column.

Requires 10 min.

4. Let

$$T = \begin{pmatrix} 1 & 3 & -2 \\ 2 & 7 & 0 \\ 1 & 1 & 3 \end{pmatrix},\quad J = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

Find the set of all solutions of the system $x' = Ax$ where $A = TJT^{-1}$

(requires 1 min since you know a basis of $\mathbb{R}^3$ consisting of eigenvalues of A).

5. Let A be a real 2×2 matrix with eigenvalue $2 - 3i$ and corresponding eigenvector $\begin{pmatrix} 5+4i \\ 3-2i \end{pmatrix}$. Find the set of all real solutions of the system $x' = Ax$. No complex numbers, signs Re, Im in the final answer.

Hint. Find a basis of complex-valued solutions consisting of vector functions $x^*(t)$ and conjugate (tsamud) $\bar{x}^*(t)$, then (as we did many times) the real and the imaginary part of $x^*(t)$ is a basis of the vector space of real solutions.

(requires 8-9 min)