ODEs - 104285. Semester: Spring. Year: 2012

HW-8. You should do it by Wednesday June 27, 16:00

1. Give an example of a 2×2 matrix A without zero entries such that the eigenvalues of A are

a) $\lambda_1 = 3,\ \lambda_2 = -4$

b) $\lambda_1 = -3,\ \lambda_2 = -4$

c) $\lambda_1 = 3i,\ \lambda_2 = -3i$

d) $\lambda_{1,2} = -2 \pm i$

and for each of these cases draw the phase portrait for the system $x' = Ax$. In the lecture I covered the phase portrait for case a) only, but the way I did it allows you to try to do it for the cases b), c), d). OK, the phase portraits in cases b), c), d) will not be in the final tests.

2. Find the set of all solutions of the system $x' = Ax$ where

a) $A = \begin{pmatrix} 3 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$

b) $A = TJT^{-1}$ where $J = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$, $T = \begin{pmatrix} 3 & 0 & 1 \\ 2 & 1 & 1 \\ 4 & 2 & 3 \end{pmatrix}$.

c) $A = \begin{pmatrix} 3 & -1 \\ 16 & -5 \end{pmatrix}$ (there is only one eigenvalue $\lambda = -1$)

3. Find e^A where A is the matrix in problem 2,c.

4. Let A be a 2×2 matrix, and let $d = detA,\ t = traceA$. Find a necessary and sufficient condition on the couple (d, t) under which any solution of the system $x' = Ax$ tends to $0 \in \mathbb{R}^2$ as $t \to \infty$.