106936: Topics in Analysis 3. HW-2

1. Find the first focus number $f_1 = f_1(a, b, c)$ for the vector field

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -x_1 + ax_1^2 + bx_1x_2 + cx_2^2$$

with parameters a, b, c.

2. Consider the vector field

$$\dot{x}_1 = x_2 + P^{(4)}(x_1, x_2) + P^{(5)}(x_1, x_2), \quad \dot{x}_2 = -x_1 + Q^{(4)}(x_1, x_2) + Q^{(5)}(x_1, x_2)$$

where $P^{(4)}$ and $Q^{(4)}$ are homogeneous degree 4 polynomials and $P^{(5)}$ and $Q^{(5)}$ are homogeneous degree 5 polynomials. The first focus number is certainly 0. Find the second focus number as a function of the coefficients of the polynomials.

3. Develop the theory of double Andronov-Hopf bifurcation for the case that the second focus number is positive. You should decompose the (ϵ_1, ϵ_2)-plane onto few regions differing by the number of limit cycles, to draw the phase portrait for (ϵ_1, ϵ_2) in each of these regions, and to explain what happens when the parameters ϵ_1, ϵ_2 change in time such that the curve $(\epsilon_1(t), \epsilon_2(t))$ crosses the boundary between the regions.