1. Homework 1. Complex numbers

1. Find all complex solutions of the equation
$$z^2 = z + 2\bar{z}$$
2. Find all $z \in \mathbb{C}$ and all $m, n \in \{1, 2, 3, ...\}$ such that
$$\big(z^m - \bar{z}^m\big)^n \in \mathbb{R}$$
3. Find all complex solutions of the equation
$$z^8 \bar{z}^3 = 1 + i$$
4. Find all $n \in \{1, 2, 3, ...\}$ such that
$$\mathrm{Re}\big((-1 - \sqrt{3} \cdot i)^n\big) > 0$$
5. Find all complex solutions of the equation
$$z^6 + 2z^3 + 3 = 0$$
6. Find exact number of complex solutions of the equation
$$z^{20} = -1 + i$$

satisfying the conditions Re $z < 0, \mathrm{Im}\ \mathrm{z} < 0$.