

**Math 106A. Fall 2008. M. Zhitomirskii**

**Homework 2. 7 problems. Due on Monday Oct 13, 9:30 am**

1. In the  $(x_1, x_2)$ -plane, draw all oriented straight line phase curves for the system  $X' = AX$  where  $X = \begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix}$  and

1.1.  $A = \begin{pmatrix} 2 & 1 \\ 5 & -2 \end{pmatrix}$     1.2.  $A = \begin{pmatrix} 2 & -1 \\ 8 & 8 \end{pmatrix}$     1.3.  $A = \begin{pmatrix} -3 & 1 \\ 4 & -3 \end{pmatrix}$

1.4.  $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$     1.5.  $A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$     1.6.  $A = \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$

2. For each of the systems 1.1 - 1.5 and each of the straight line phase curve, find a solution  $X = \begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix}$  corresponding to this phase curve.

3. Find all values of the parameter  $a \in \mathbb{R}$  such that the system  $X' = AX$  with  $X = \begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix}$  and

3.1.  $A = \begin{pmatrix} 1 & a \\ 1 & 4 \end{pmatrix}$      $A = \begin{pmatrix} a & 1 \\ 1 & 1 \end{pmatrix}$

has at least one straight line phase curve.

4. In the  $(x, x')$ -plane, draw all straight line phase curves for each of the following equations:

4.1.  $x'' - 4x' + 3x = 0, \quad x = x(t)$

4.2.  $x'' - 3x' - 4x = 0, \quad x = x(t)$

4.3.  $x'' + 7x' + 10x = 0, \quad x = x(t)$

5. Let  $A$  be one of the matrices given below. How many straight line phase curves does the system  $X' = AX$  have? Here  $X = \begin{pmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \end{pmatrix}$ .

5.1.  $A = \begin{pmatrix} 0 & 2 & 5 \\ 5 & 3 & 8 \\ 0 & 0 & 1 \end{pmatrix}$     5.2.  $A = \begin{pmatrix} 0 & 2 & 0 \\ 5 & 3 & 0 \\ -1 & 4 & 5 \end{pmatrix}$

5.3.  $A = \begin{pmatrix} 3 & 2 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix}$     5.4.  $A = \begin{pmatrix} 3 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

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6. For each of the matrices  $A$  given in problems 1.1 - 1.4 find the solution  $X = \begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix}$  of the system  $X' = AX$  satisfying the initial condition

a.  $X(0) = \begin{pmatrix} 5 \\ 9 \end{pmatrix}$       b.  $X(-3) = \begin{pmatrix} 5 \\ 9 \end{pmatrix}$

7. For each of the matrices  $A$  given in problems 5.1 - 5.2 find the solution  $X = \begin{pmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \end{pmatrix}$  of the system  $X' = AX$  satisfying the initial condition

a.  $X(0) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$       b.  $X(0) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$