106803. Homework 2. Deadline: March 23.

1. For which integer N the couple of vector fields on $\mathbb{R}^3$

$$V_1 = \frac{\partial}{\partial x_1}, \quad V_2 = \frac{\partial}{\partial x_2} + x_1^N \frac{\partial}{\partial x_3}$$

is bracket generating?

2. Find the condition on the parameters a_1, a_2, a_3 and b_1, b_2, b_3 under which the couple of vector fields on $\mathbb{R}^5$

$$V_1 = \frac{\partial}{\partial x_1} + x_2 \frac{\partial}{\partial x_3} + (a_1 x_1^2 + a_2 x_1 x_2 + a_3 x_2^2) \frac{\partial}{\partial x_4} + x_2^2 \frac{\partial}{\partial x_5}$$

$$V_2 = \frac{\partial}{\partial x_2} - x_1 \frac{\partial}{\partial x_3} - x_1^2 \frac{\partial}{\partial x_4} + (b_1 x_1^2 + b_2 x_1 x_2 + b_3 x_2^2) \frac{\partial}{\partial x_5}$$

is NOT bracket generating.

3. Find the condition on the parameters b_{ij} under which the couple of vector fields on $\mathbb{R}^3$

$$V_1 = x_1 \frac{\partial}{\partial x_1} + 2x_2 \frac{\partial}{\partial x_2} + 3x_3 \frac{\partial}{\partial x_3}$$

$$V_2 = \Big(b_{11}x_1 + b_{12}x_2 + b_{13}x_3\Big) \frac{\partial}{\partial x_1} + \Big(b_{21}x_1 + b_{22}x_2 + b_{23}x_3\Big) \frac{\partial}{\partial x_2} + \Big(b_{31}x_1 + b_{32}x_2 + b_{33}x_3\Big) \frac{\partial}{\partial x_3}$$

is integrable.