106803. Homework 3. Deadline: April 29.

1. Prove that the second coordinate of the growth vector of a $(3, n)$ distribution does not depend on the basis of vector fields used in the definition of the growth vector.

2. Prove that the third coordinate of the growth vector of a $(2, n)$ distribution does not depend on the basis of vector fields used in the definition of the growth vector.

3. Give an example of a $(2, 5)$ distribution with the growth vector $(2, 3, d_3, d_4, ...)$ where $d_i = 4$ for any $i \geq 3$ at any point of $\mathbb{R}^5$ (such distribution is not bracket generating).

4. Give an example of a bracket generating $(2, 4)$ distribution with the growth vector at $0 \in \mathbb{R}^4$

(a) $(2, 2, 2, 4)$

(b) $(2, 3, 3, 4)$

5. Prove the formula $[V_1, V_2](f) = V_1(V_2(f)) - V_2(V_1(f))$ (here V_1, V_2 are vector fields, f is a function).

6. Write down the vector field $\Bigg[V_1, \Big[[V_1, V_2], \Big[V_1, [V_1, V_2]\Big]\Big]\Bigg]$ as a linear combination of the vector fields of the form $[V_{i_1}, [V_{i_2}, [V_{i_3}, , [\cdots [V_{i_m}]] \cdots]$ where $i_1, ..., i_m \in \{1, 2\}$.

7. Compute (using the formula $[V_1, fV_2] = V_1(f)V_2 + f[V_1, V_2]$)

$$\Big[(x_1^2 + x_2^2)V_1 + x_1x_3V_2 + x_2x_3V_3,\ x_1x_2V_1 + x_1x_2x_3V_2 + x_3^2V_3\Big],$$

where

$$V_1 = \frac{\partial}{\partial x_1}, \quad V_2 = \frac{\partial}{\partial x_2} + x_1\frac{\partial}{\partial x_3}, \quad V_3 = x_2\frac{\partial}{\partial x_3}.$$