1. Homework 3. Systems of linear equations: part 1
Solve problems 1.2, 2.2, 3.2, 4.2, 4.4, 5.2, 5.4, 5.6, 5.8. (The other problems: for tirgul).

1. Write the following systems in the form $Ax = b$, where A is a matrix and b is a vector

1.1. $x_2 - x_3 = 1, \quad x_1 + x_2 = 2, \quad x_2 - x_4 = 0$

1.2. $x_2 + x_3 = 0, \quad x_3 - x_1 = 0, \quad x_1 + x_2 + 2x_3 = 1, \quad x_2 + 4x_3 = 2.$

2. Let

$$A = \begin{pmatrix} 4 & i & 2 & 0 \\ 2 & 0 & i & 0 \\ 3 & 4 & 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 0 & 4 \\ 1 & 2 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 3 & 0 \\ 4 & 1 & 1 \end{pmatrix}, \quad x = \begin{pmatrix} 1 \\ 2 \\ 0 \\ -1 \end{pmatrix}$$

2.1. Find $C \cdot (B \cdot (Ax))$.   2.2. Find $B \cdot (C \cdot (Ax))$.

3.1. Let A be a 3×3 matrix such that $A_{ij} = 0$ if $i \geq j$ and let $x \in \mathbb{R}^3$. What can you say about the vector $A \cdot x$? About the vector $A \cdot (Ax)$? About the vector $A \cdot (A \cdot (Ax))$?

3.2. Let A be a 4×4 matrix such that $A_{ij} = 0$ if $i \leq j$. Prove that $A \cdot (A \cdot (A \cdot (Ax))) = 0$ for any $x \in \mathbb{R}^4$.

4. LEDAREG the following matrices (with parameter $a \in \mathbb{R}$) and find their rank. The rank might depend on a.

4.1. $\begin{pmatrix} 2 & 1 & 2 & 4 \\ 3 & 0 & 0 & 6 \\ a & 0 & 0 & 8 \end{pmatrix}$ 4.2. $\begin{pmatrix} 2 & 1 & 0 & 5 \\ 3 & 0 & 1 & 7 \\ 6 & 0 & 2 & a \end{pmatrix}$ 4.3. $\begin{pmatrix} 2 & 1 & 0 \\ 4 & 2 & 1 \\ 2 & 1 & a \\ 4 & 2 & a \end{pmatrix}$ 4.4. $\begin{pmatrix} 2 & 1 & 0 \\ 4 & 2 & 1 \\ 2 & 1 & 3 \\ a & 3 & 0 \end{pmatrix}$

5. For which $a, b \in \mathbb{R}$ the rank of the following matrices is equal (a) to 1 (b) to 2 (c) to 3?

5.1. $\begin{pmatrix} 2 & 3 & 1 \\ 3 & a & b \end{pmatrix}$ 5.2. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ a & b & 3 \end{pmatrix}$ 5.3. $\begin{pmatrix} a & 0 & 1 \\ 0 & 0 & b \\ 1 & 1 & 1 \end{pmatrix}$

5.4. $\begin{pmatrix} 0 & a & b \\ -a & 0 & 0 \\ -b & 0 & 0 \end{pmatrix}$ 5.5. $\begin{pmatrix} 1 & a \\ b & 1 \\ 2 & 0 \end{pmatrix}$ 5.6. $\begin{pmatrix} a & 1 & 0 & 1 \\ 0 & 1 & 0 & b \\ 2 & 0 & 1 & 0 \end{pmatrix}$

5.7. $\begin{pmatrix} a^2 - 1 & 1 & 0 & 2 \\ a^2 - 2 & 1 & 1 & 0 \\ b & 1 & 0 & 0 \end{pmatrix}$ 5.8. $\begin{pmatrix} a & b \\ -a & 2b+1 \end{pmatrix}$