

Math 106A. Fall 2008. M. Zhitomirskii
Homework 4. 3 problems. Due on Friday Oct 31, 9:30 am

1. Consider the system $x'_1 = x_1 - 2x_2$, $x'_2 = ax_1 + 3x_2$ with a parameter $a \in \mathbb{R}$.

1.1. For which values of a the phase portrait is a source? sink? saddle? spiral source? spiral sink? center? contains a line of equilibrium points?

1.2. Let $a = 13$. Draw the oriented phase portrait. Do not forget to determine whether the phase curves go clockwise or anticlockwise. Find the natural period of oscillations.

1.3. Let $a = 13$. Find the solution satisfying the initial conditions $x_1(0) = x_2(0) = 1$. For this solution, find all t such that $x_1(t) = 0$ and all t such that $x_2(t) = 0$.

1.4. Draw the phase portrait for $a = 0$.

1.5. Draw the phase portrait for $a = -2$.

2. Consider the system $x'_1 = 3x_1 + ax_2$, $x'_2 = 13x_1 - 3x_2$ with a parameter $a \in \mathbb{R}$.

2.1. For which values of a the phase portrait is a source? sink? saddle? spiral source? spiral sink? center? contains a line of equilibrium points?

2.2. Let $a = -1$. Draw the oriented phase portrait. Do not forget to determine whether the phase curves go clockwise or anticlockwise. Find the period of oscillations.

2.3. Let $a = -1$. Find the solution satisfying the initial conditions $x_1(0) = x_2(0) = 1$. For this solution, find all t such that $x_1(t) = x_2(t)$. Draw the graphs of the function $x_1(t)$ and $x_2(t)$.

3. Consider the system $x'_1 = 3x_1 - x_2$, $x'_2 = 13x_1 + ax_2$ with a parameter $a \in \mathbb{R}$.

3.1. For which values of a the phase portrait is a source? sink? saddle? spiral source? spiral sink? center? contains a line of equilibrium points?

3.2. Let $a = -4$. Draw the oriented phase portrait. Do not forget to determine whether the phase curves go clockwise or anticlockwise. Find the natural period of oscillations.