Homework 4. Systems of linear equations: part 2
Assignment: solve all problems including those which will be solved in tirgulim

Notations: $\exists$ = exists, $\forall$ = for all = for any.

1. Which of the following statements are right for any 3×4 matrix A?

1.1 rank $A < 4$

1.2. If rank $A = 3$ then the system $Ax = b$ has unique solution $\forall b \in \mathbb{R}^3$

1.3. If rank $A = 2$ then the system $Ax = b$ has ∞ solutions $\forall b \in \mathbb{R}^3$

1.4. The system $Ax = 0$ has ∞ solutions

2. Which of the following statements are right for any 4×3 matrix A?

2.1. rank $A < 4$

2.2. If rank $A = 3$ then the system $Ax = b$
has at least one solution $\forall b \in \mathbb{R}^4$

2.3. If rank $A = 2$ then $\exists b \in \mathbb{R}^4$ such that
the system $Ax = b$ has unique solution

2.4. If $A \neq 0$ then the system $Ax = 0$ has unique solution.

3. In each of the 6 statements given below
A is an $m \times n$ matrix and $b \in \mathbb{R}^m$.
Which of the statements are WRONG in the cases
(a) $m = n$ (b) $m < n$ (c) $m > n$?
Note that the problem contains $3 \times 6 = 18$ questions.

A1. $\exists A$ and $\exists b$ such that the system $Ax = b$ has unique solution.

B1. $\exists A$ such that the system $Ax = b$ has unique solution $\forall b$.

A2. $\exists A \neq 0$ and $\exists b \neq 0$ such that the system $Ax = b$ has no solutions.

B2. $\exists A \neq 0$ such that the system has no solutions $\forall b \neq 0$.

A3. $\exists A \neq 0$ and $\exists b \neq 0$ such that the system $Ax = b$ has ∞ solutions.

B3. $\exists A$ such that the system $Ax = b$ has ∞ solutions $\forall b$.

4. Find conditions on the parameters $a, b \in \mathbb{R}$ such that the following system has
(a) no solution (b) unique solution (c) ∞ solutions

4.1. $2x_1 + 3x_2 + 5x_3 = 10,\ \ ax_2 + x_3 = b,\ \ 2x_2 + 3x_3 = 1$

4.2. $2x_1 + 3x_2 + 5x_3 = 10,\ \ 7x_2 + 8x_3 = 12,\ \ ax_1 + x_2 = b$

4.3. $ax_1 + x_2 = 1,\ \ 2x_1 + x_3 = b,\ \ x_1 + x_2 - x_3 = 0$

4.4. $x_1 + x_2 - 2x_3 = 0,\ \ ax_1 - x_2 + x_3 = 1,\ \ bx_1 - x_2 - x_3 = 0$

4.5. $x_1 + x_2 - x_4 = 0,\ \ x_2 - x_3 = 1,\ \ x_1 + ax_3 + bx_4 = 1$

4.6. $x_1 - x_2 + x_3 = 1,\ \ 2x_1 - x_2 - x_3 = a,$
$x_1 + 2x_2 - 3x_3 = b,\ 2x_1 - x_2 - x_3 = 1$