Math 106A. Fall 2008. M. Zhitomirskii
Homework 5. 1 problem. Due on Wednesday, November 5, 9:30 am

Let A be a real 8×8 matrix with eigenvalues

$$\lambda_1 = 2 + 5i, \lambda_2 = -3 + 4i,\ \lambda_3 = 7,\ \lambda_4 = -8,\ \lambda_5 = 10i$$

and corresponding eigenvectors

$$v_1 = \begin{pmatrix} 4 - 9i \\ 8 + 11i \\ 10 + 7i \\ 4 - 13i \\ 2 \\ 3 + 2i \\ 14 - 5i \\ 10 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 6 \\ 10 + i \\ 11 \\ 14 + 2i \\ 9 - 4i \\ 1 + i \\ 0 \\ 0 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 0 \\ 0 \\ 12 \\ 7 \\ 1 \\ 0 \\ 19 \\ 8 \end{pmatrix}, \quad v_4 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 6 \\ -3 \\ 8 \\ 0 \end{pmatrix},$$

$$v_5 = \begin{pmatrix} 1 - 2i \\ 3 + 9i \\ i \\ 5 - i \\ 1 \\ 0 \\ 8 + 7i \\ 11 - i \end{pmatrix}$$

Let $X(t)$ be the solution of the system $X' = AX$ satisfying the initial condition

$$X(0) = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{pmatrix}$$

where $a_1, ..., a_8$ are real numbers. Find a necessary and sufficient condition on $a_1, ..., a_8$ such that

(a) $X(t)$ is a periodic vector function

(b) $X(t) \to 0$ as $t \to \infty$

(c) $X(t) \to 0$ as $t \to -\infty$

Take any $a_1, ..., a_8$ satisfying (a) so that the solution $X(t)$ is periodic. Find $max|x_1(t)|,\ max|x_2(t)|, ..., max|x_8(t)|$.