Homework 5. Deadline: Thursday, June 16

1. Find any curve tangent to the distribution $span(\partial/\partial x_1,\ \partial/\partial x_2 + x_1 \partial/\partial x_3)$ which joins $(0,0,0)$ to $(1,1,1)$ and find the length of this curve in the sub-Riemannian metrics defined by the condition that $\partial/\partial x_1,\ x_1\partial/\partial x_1 + \partial/\partial x_2 + x_1\partial/\partial x_3$ is the orthonormal basis of vector fields.

2. Consider the control system $\dot{x}_1 = u,\ \dot{x}_2 = x_1$ where $u = u(t)$ is the control (t is the time). Let T^* be the infimum of time which one needs to join $(0,0)$ to $(1,-1)$ under constraint

(1) $$|x_1(t)| \leq 10, |x_2(t)| \leq 10.$$

Find T^* and for any positive ϵ find a curve which satisfies (1) and joins $(0,0)$ to $(1,-1)$ in time $T < T^* + \epsilon$. Draw this curve in the (x_1, x_2)-plane.

3. Solve problem 2 under constraint $|u(t)| \leq 10$ instead of (1).