Math 106A. Fall 2008. M. Zhitomirskii
Homework 8. 3 problems. Due on Friday, December 5, 9:30 am

1. Find all values of the parameters $a, b, c \in \mathbb{R}$ such that the singular point $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ of the following system is asymptotically stable:

1.1. $\dot{x}_1 = x_2, \quad \dot{x}_2 = ax_1 + bx_2 + cx_2^2$

1.2. $\dot{x}_1 = asin(x_1) + cos(x_2) - 1, \quad \dot{x}_2 = e^{bx_1 + cx_2} - 1$

1.3. $\dot{x}_1 = ln(x_1^2 + x_2^2 + x_1 + x_2 + 1), \quad \dot{x}_2 = ax_1 + bx_2$

2. Find ALL singular (= equilibrium) points of the following systems and for each of them find all values of the parameters $a, b \in \mathbb{R}$ such that this singular point is asymptotically stable:

2.1. $\dot{x}_1 = x_1^2 + x_2^2 - 1, \quad \dot{x}_2 = ax_1 + bx_2$

2.2. $\dot{x}_1 = sin(x_1 + x_2), \quad \dot{x}_2 = x_1 - ax_2$

2.3. $\dot{x}_1 = sin(x_1 + x_2), \quad \dot{x}_2 = asin(x_1 - x_2), \quad a \neq 0$

3. Give an example of a function $f(x_1)$ such that the system

$$x_1' = x_2, \ x_2' = f(x_1) + x_1 x_2$$

has an asymptotically stable equilibrium point.