

**Math 106A. Fall 2008. M. Zhitomirskii**  
**Homework 9 (preparation to the final). 4 problems.**

1. Find the solution of the system

$$x'_1 = -x_1 + 2x_2 + \sin t, \quad x'_2 = -13x_1 - 3x_2 + \cos t$$

satisfying the initial conditions

$$x_1(0) = 1, \quad x_2(0) = 0.$$

2. Find the solution of the equation

$$y'' - 3y' + 4y = \sin t$$

satisfying the initial condition

$$y(0) = 0, \quad y'(0) = 1.$$

3. Consider the system

$$x'_1 = x_1 + 2x_2 + \epsilon \cdot x_2^2, \quad x'_2 = -3x_1 - x_2 + \epsilon x_1^3$$

with a small parameter  $\epsilon$ . Let

$$x_1 = x_{1,0}(t) + \epsilon x_{1,1}(t) + h.o.t., \quad x_2(t) = x_{2,0}(t) + \epsilon x_{2,1}(t) + h.o.t.,$$

where h.o.t. denotes non-linear terms with respect to  $\epsilon$ , be the solution of this system satisfying the initial condition

$$x_1(0) = 1 - 3\epsilon, \quad x_2(0) = 3 + \epsilon.$$

Find  $x_{1,0}(t), x_{2,0}(t)$  and set up a system of equations for  $x_{1,1}(t)$  and  $x_{2,1}(t)$ .

4. Let

$$y = y_0(t) + \epsilon y_1(t) + h.o.t.,$$

where h.o.t. denotes non-linear terms with respect to  $\epsilon$ , be the solution of the equation

$$y'' + 6y' + 12y = \epsilon \cdot y \cdot y'$$

with a small parameter  $\epsilon$  satisfying the initial conditions

$$y(0) = 1, \quad y'(0) = 0.$$

Find  $y_0(t)$  and  $y_1(t)$ .