Math 106A. Fall 2008. M. Zhitomirskii
Exam 2. November 17, 2008

Three problems. In the first problem the eigenvalues are $-2 \pm i$ and $\pm 2i$. The computations (linear algabra, complex numbers) will take from 15 to 30 min. The second problem requires 5 to 10 min if you use the following fact: if A, B, C are 2×2 matrices then the the 4×4 matrix $\begin{pmatrix} A & C \\ 0 & B \end{pmatrix}$ and $\begin{pmatrix} A & 0 \\ C & B \end{pmatrix}$ have the same determinant $detA \cdot detB$. The third problem requires 15 to 30 min. The three parts in this problem correspond to different types of the phase portraits. The bonus part is the phase portrait in case c). This is what we did not do in the class, but you know everything required to do it. **Good luck!**

NAME (FIRST, LAST) _____

1.a : Find b_1, b_2		11 pts
1.a : Find $X(t)$		5 pts
1.b : Find b_1, b_2		11 pts
1.b : Find $X(t)$		5 pts
2.a		6 pts
2.b		7 pts
3.a: phase portrait		11 pts
3.a: solution		11 pts
3.b: phase portrait		11 pts
3.b: solution		11 pts
3.c: phase portrait (**this is a bonus problem**)		10 pts
3.c: solution		11 pts
TOTAL		110 pts

1. Let $X(t)$ be the solution of the system

$$X' = \begin{pmatrix} 1 & 2 & 0 & 0 \\ -5 & -5 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 5 & -1 \end{pmatrix}$$

satisfying the initial condition

$$X(0) = \begin{pmatrix} 1 \\ 1 \\ b_1 \\ b_2 \end{pmatrix}.$$

Find b_1, b_2 and find $X(t)$ if it is knows that

(a) $X(t) \to 0 \in \mathbb{R}^4$ as $t \to \infty$

(b) $X(t)$ is a periodic vector-function.

2. Give an example of a *real* 4×4 matrix with eigenvalue $1 + i$ of algebraic multiplicity 2 and

(a) geometric multiplicity 1

(b) geometric multiplicity 2

3. Consider the system

$$x_1' = 3x_1 + a \cdot x_2, \quad x_2' = x_1 - 3x_2$$

(a) Let $a = -8$. Draw the oriented phase portrait. Make bold the oriented phase curve corresponding to the initial condition

$$x_1(0) = x_2(0) = 1$$

Find the solution satisfying this initial condition.

(b) All tasks in (a) for $a = -10$

(c) All tasks in (a) for $a = -9$.