MATH 23B: Multivariable Calculus

Midterm

Winter 2009

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SHOW ALL WORK!

Problem 1 ________________ (40 pts.)

Problem 2 ________________ (40 pts.)

Problem 3 ________________ (40 pts.)

TOTAL ________________ (120 pts.)

NAME (Last, First) ______________________________

STUDENT NUMBER: __________________

I WANT TO GET BACK MY GRADED WORK
ON THE DISCUSSION SECTION (MARK ONE OF THE BOXES):

☐ by Ted Nitz (T/Th 8:00 - 9:45 am and 2:00 - 3:45 pm)

☐ by Wyatt Howard (T/Th 6:00 - 7:45 pm, MW 7:00 - 8:45 pm)

SIGNATURE: ________________________

You are allowed to use up to four handwritten pages with any notes. Two sides is OK. Books, laptops, and calculators (even simplest) are not allowed.

GOOD LUCK!

Problem 1. Compute

$$\int\int_D \frac{x}{y}\, dA,$$

where D is the parallelogram with vertices at

$$(2, 1),\ (4, 1),\ (3, 2),\ \text{and } (5, 2).$$

Problem 2.

(a) Sketch the region $D \subset \mathbb{R}^2$ such that

$$\int_0^{2/3} \int_0^x f(x,y)dydx + \int_{2/3}^1 \int_0^{2(1-x)} f(x,y)dydx = \int\int_D f(x,y)dA$$

(for any continuous function $f(x,y)$).

HINT: $D = D_1 \cup D_2$ where D_1 and D_2 are the regions determined by the limits in the first and the second iterated integrals.

(b) Using (a), find numbers α, β and functions $\phi_1(y), \phi_2(y)$ such that

$$\int_0^{2/3} \int_0^x f(x,y)dydx + \int_{2/3}^1 \int_0^{2(1-x)} f(x,y)dydx = \int_\alpha^\beta \int_{\phi_1(y)}^{\phi_2(y)} f(x,y)dxdy$$

(for any continuous function $f(x,y)$).

Problem 3. Set up (but do not compute) an iterated integral for the volume of the region $W \subset \mathbb{R}^3$ determined by the conditions

$$W = \{(x, y, z) : \ 3z^2 \le x + 2y \le z, \quad x \ge 0, \quad y \ge 0\}.$$

HINT. Find the projection of W to the (x, y)-plane, i.e. the region

$$D = \{(x, y) : \quad \text{there exits } z \text{ such that } (x, y, z) \in W\} \subset \mathbb{R}^2.$$

If you sketch the region D, it will be easy to describe W as an elementary region (start with describing D as an elementary region).
