Tirgul: Complex roots of polynomials with real coefficients

1. Find all complex roots of the polynomial

$$P(z) = (z^4 - 16)^3 \cdot (z^3 - 1)^4.$$

Find the multiplicity of each of the roots.

2. It is known that the polynomial $P(z) = z^4 + a_3z^3 + a_2z^2 + a_1z + a_0$ with real coefficients a_0, a_1, a_2, a_3 has a real root $z = 5$ and there are no other real roots. It is also known that $P(z)$ has a root $z = i$. Find a_0, a_1, a_2, a_3.

3. Find real positive number a such that the polynomial

$$P(z) = az^3 - z + 1$$

has less than 3 complex roots.

4. Find $a_1, a_2, a_3 \in \mathbf{R}$ such that the polynomial

$$P(z) = z^4 + a_3z^3 + a_2z^2 + a_1z + 1$$

has a root $z = -1$ of multiplicity 2.

5. It is known that the polynomial

$$P(z) = z^5 + a_4z^4 + a_3z^3 + a_2z^2 + a_1z + 1$$

with real coefficients a_1, a_2, a_3, a_4 has a root $z = 1 + i$ of multiplicity 2. Find a real root of $P(z)$. Find a_4.