Problem 1. Find the area of the region D in the (x, y)-plane which is obtained from the rectangle $[0, 1] \times [0, 1]$ in the (u, v)-plane by the transformation $x = u^2 + u + v,\ y = 3v$.

Problem 2. Find the volume of the region D in the (x, y, z)-space which is obtained from the unit ball $W : \{u^2 + v^2 + w^2 \leq 1\}$ in the (u, v, w)-space by the transformation
$x = u + v, y = u - v, z = 3u - 4v + 5w$
(you can use the fact that the volume of W is equal to $4\pi/3$).

Problem 3. Give an example of a function $g(x, y)$ such that

$$\int_c (2x^3y + x^2y^2)dx + g(x, y)dy = 0$$

for any closed path c.

Problem 4. Give an example of a function $g(x, y)$ such that the flux of the vector field $\mathbf{F} = (2x^3y + x^2y^2)\mathbf{i} + g(x, y)\mathbf{j}$ out of any closed curve is equal to 0.

Problem 5. Give an example of two functions $h_1(x, y, z), h_2(x, y, z)$ such that the integral of the vector field $\mathbf{F} = xyz\mathbf{i} + h_1(x, y, z)\mathbf{j} + h_2(x, y, z)\mathbf{k}$ over any closed path is equal to 0.

Problem 6. Find a function $f(\theta)$ such that the length of the closed curve $C : x^2 + xy + 4y^2 = 0$ is equal to $\int_0^{2\pi} f(\theta)d\theta$. (Hint: C is an ellipse, it can be transformed to a circle by a linear transformation). Find the area bounded by C (the answer must be a number).

Problem 7. Compute the integral of the vector field $\mathbf{F} = xy\mathbf{i} + yz\mathbf{i} + \mathbf{k}$ over the circumference of the traingle with vertices $(0, 0, 0)$, $(2, 1, 3)$, and $(2, 4, 3)$.

Problem 8. Use Green's theorem to compute $\int\int_D x^2 dS$, where D is the region between by the circle $x^2 + y^2 = 100$ and the ellipse
$2x^2 + 4y^2 = 1$.

Problem 9. The surface S is determined by the conditions
$x^2 + y^2 \leq 4, z = xy$. Find the unit normal vector to S at the point $(\sqrt{2}/2, \sqrt{2}/2, 1/2)$. Set up an iterated integral for the area of S.

Problem 10. Use Gauss' theorem to find the flux of the vector field $\mathbf{F} = x^2\mathbf{i} + xz\mathbf{j} + y^2z\mathbf{k}$ out of the

(a) surface of the cube $[0, 1] \times [0, 1] \times [0, 1]$

(b) the sphere $x^2 + y^2 + z^2 = 1$.

(c) the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$.