

106936: Topics in Analysis 3. Projects.

1. Develop the theory of triple Andronov-Hopf bifurcation: the case that a vector field depends on 3 parameters and for certain value of the parameters we have pure imaginary eigenvalues and zero first and the second focus numbers. You should consider the cases of both negative and positive third focus number. The theory should contain pictures on the birth and death of limit cycles.

2. Consider a vector field of the form

$$\dot{x}_1 = x_2 + a_1x_1^3 + a_2x_1^2x_2 + a_3x_1x_2^2 + a_4x_2^3, \quad \dot{x}_2 = -x_1 + b_1x_1^3 + b_2x_1^2x_2 + b_3x_1x_2^2 + b_4x_2^3.$$

Write a computer program with input $a_1, \dots, a_4, b_1, \dots, b_4$ which computes the first focus number f_1 , stops if $f_1 \neq 0$, computes the second focus number f_2 , stops if $f_2 \neq 0$, and computes the third focus number f_3 if $f_1 = f_2 = 0$. The program should give a conclusion whether we have a stable or unstable focus or to inform that the first three focus numbers are zero and the phase portrait depends on the next focus numbers.